Sensor Fault Detection, Isolation, and Estimation in Lithium-Ion Batteries

Satadru Dey, Sara Mohon, Pierluigi Pisu, and Beshah Ayalew

Abstract—In battery management systems (BMS), real-time diagnosis of sensor faults is critical for ensuring the safety and reliability of the battery. For example, a current sensor fault leads to erroneous estimates of state of charge and other parameters, which in turn affects the control actions in the BMS. A temperature sensor fault may lead to ineffective thermal management. In this brief, a model-based diagnostic scheme is presented that uses sliding mode observers designed based on the electrical and thermal dynamics of the battery. It is analytically shown how the extraction of the equivalent output error injection signals on the sliding manifolds enables the detection, the isolation, as well as the estimation of the temperature, voltage, and current sensor faults. This brief includes simulation and experimental studies to demonstrate and evaluate the effectiveness of the proposed scheme. Discussions are also included on the effects of uncertainty and on threshold design.

Index Terms—Fault detection and isolation, fault estimation, Li-ion batteries, sensor fault, sliding mode observers.

I. INTRODUCTION

E STIMATION and control algorithms in battery management systems (BMSs) heavily depend on the real-time measurements of battery voltage, current, and temperature. Any fault in these sensors could hinder the BMS operation and lead to catastrophic scenarios. However, in the existing Li-ion battery literature, while several model-based state estimation algorithms are presented [1]–[4], the real-time fault diagnosis problem is much less explored.

The reviews of failure mechanisms and the diagnostics related challenges can be found in [5] and [6]. In [7], a nonlinear fault detection and isolation scheme has been developed to detect sensor and actuator faults. A Kalman-filter-based scheme was proposed in [8] to detect overcharge and overdischarge faults. A universal adaptive stabilization technique was proposed in [9] for diagnosing terminal voltage collapses. In [10], a set of Luenberger and learning observers were used for simultaneous fault isolation and estimation of a faulty cell in a battery string. In [11], diagnostic algorithms are presented for a battery pack. A fault detection and isolation strategy has been presented in [12] based on structural analysis.

Most of these existing approaches detect and/or isolate some faults in the battery. However, very few of them actually estimate (some characteristics of) specific sensor faults. In this brief, we fill this gap and extend the aforementioned research by proposing a real-time diagnostic approach that detects, isolates, and estimates specific sensor faults, namely, the voltage, temperature, and current sensor faults. Note that completing the detection and isolation steps alone may indicate the fault occurrence and identify the faulty sensor, but sensor fault estimation can provide crucial added benefits for enhancing the reliability of BMSs. For example, current sensor fault information can help augment or correct the popular coulomb-counting-based state-of-charge (SOC) computation. Furthermore, current and temperature sensor fault information can help correct state-of-health parameter estimation schemes, which generally depend on SOC and temperature. Especially, in electric vehicle applications, it will enable the limp-home mode by allowing the vehicle to be driven safely to home or to a repair shop even after the occurrence of the fault. Broadly, sensor fault estimation can provide fault-tolerance capability to the BMS by allowing it to continue to function (although possibly in a degraded but safe mode) even after the sensor fault occurs.

In this brief, a diagnostic approach is presented that estimates the sensor faults along with their detection and isolation. As our focus is on output (sensor) faults, a model of the battery cell comprising of an equivalent electrical circuit coupled to a lumped thermal dynamics model is adopted for predicting the cell output behavior for its simple structure and low computational burden. The work adopts and builds on a sliding mode observer-based fault detection methodology presented in [13]. The basic idea to be exploited is that the equivalent output error injection, which is a continuous approximation (or filtered version) of the switching error injection term in the sliding mode observer, captures the fault information on the sliding surface. In this brief, three sliding mode observers are designed using the electrical plus thermal model. Then, on the sliding surfaces of each observer, a set of fault detection filter expressions are derived that are driven by the equivalent output error injections from the observers. The outputs of these filters are used as residual signals to detect, isolate, and estimate the sensor faults under the assumption that the faults and their time derivatives are bounded and finite. Analytical justifications of the proposed scheme are given using Lyapunov analysis for each fault scenario.

The rest of this brief is organized as follows. Section II briefly describes the battery cell model adopted for this work, and Section III briefs the diagnostics problem and outlines the detail design of the fault diagnosis scheme. Section IV presents discussions of some simulation and
are given by the lumped thermal dynamics of the cell and the sensor faults, respectively. It is assumed that the parameters are constant and known with sufficient accuracy.

II. LITHIUM-ION CELL MODEL

For a model of a battery cell, the commonly used equivalent electrical circuit (Fig. 1) plus the lumped thermal dynamics model are adopted.

The electrical dynamics of the battery cell can be written using Kirchoff’s law, and with a usual definition of SOC

\[ \dot{V}_c = -\frac{V_c}{R_0 C_0} + \frac{I}{C_0} \]  
\[ \dot{\text{SOC}} = -\frac{I}{Q} \]  
\[ V = E_0 - IR - V_c \]

where \( V \) is the terminal voltage, \( I \) is the input current, \( R \) and \( R_0 \) are the resistors and capacitors of the electrical circuit, respectively, \( V_c \) is the voltage across the capacitor \( C_0 \), \( E_0 \) is the open-circuit voltage, and \( Q \) is the charge capacity of the battery cell. The lumped thermal dynamics of the cell are given by

\[ mc \dot{T} = I^2 (R + R_0) - hA(T - T_{\text{amb}}) \]  

where \( T \) is the battery cell temperature, \( m \) is the mass, \( c \) is the specific heat capacity of the battery cell, \( hA \) is the effective heat transfer coefficient, and \( T_{\text{amb}} \) is the ambient temperature. Note that the open-circuit voltage is a function of SOC

\[ E_0 = f(\text{SOC}) \]  

In the presence of sensor faults, the sensor outputs can be modeled by

\[ I_{\text{meas}} = I + \Delta I, \quad V_{\text{meas}} = V + \Delta V, \quad T_{\text{meas}} = T + \Delta T \]

where \( I_{\text{meas}}, V_{\text{meas}}, \) and \( T_{\text{meas}} \) are the measured variables and \( \Delta I, \Delta V, \) and \( \Delta T \) are the current, voltage, and temperature sensor faults, respectively. It is assumed that \( \Delta I \) and \( \Delta V \) are bounded by the finite values \( |\Delta I|_{\text{max}} \) and \( |\Delta V|_{\text{max}} \), respectively, where \( k \in \{I, V, T\} \). It is also assumed that no multiple faults can occur at the same time. Furthermore, we consider the case of small SOC ranges (valid for applications such as hybrid electric vehicles), where the parameters \( R, R_0, C_0, mc, \) and \( hA \) are constant and known with sufficient accuracy.

III. DIAGNOSTIC PROBLEM AND PROPOSED SCHEME

In this brief, we mainly focus on the faults in the sensors: current sensor, voltage sensor, and temperature sensor. The impact of these faults in BMS operation is discussed in [12].

A. Diagnostic Scheme

The diagnostic scheme is shown in Fig. 2. In the following paragraphs, the elements of the scheme are described in detail.

1) Observers: In our scheme, we use three observers, one of which is based on (equivalent circuit) capacitor voltage \( V_c \) dynamics given in (1) \([V_c-Observer in (7) and (8)]\) and the other two are based on temperature dynamics given in (4) \([T-Observer 1 in (9) and T-Observer 2 in (10)]\). From these observers, we extract the equivalent output error injection signals required to maintain the sliding motion [14].

a) \( V_c \)-Observer:

\[ \dot{\hat{V}_c} = -\frac{\hat{V}_c}{R_0 C_0} + \frac{I_{\text{meas}}}{C_0} + L_v \text{sgn}(V_{c-\text{meas}} - \hat{V}_c) \]  
\[ V_{c-r} = E_0 - I_{\text{meas}} R - V_{\text{meas}} \]

where \( V_{c-r} \) is the reconstructed value of \( V_c \), \( L_v > 0 \) is the tunable observer gain, and \( E_0 \) is calculated via \( E_0 = f(\text{SOC}) \), where \( \hat{\text{SOC}} = -\frac{I_{\text{meas}}}{Q} \text{sgn}(\hat{V}_c - \hat{V}_c) \) assuming \( Q \) is known with sufficient accuracy. Note that the initial SOC can be estimated from the \( E_0 - \text{SOC map} f(\cdot) \) when the cell is at rest for a sufficient amount of time. However, some amount of SOC error and hence the \( E_0 \) error can be treated as uncertainties and suppressed by the nonzero thresholding scheme discussed in Section III-C.

b) \( T \)-Observer 1:

\[ mc \dot{\hat{T}_1} = I_{\text{meas}}^2 (R + R_0) - hA(\hat{T}_1 - T_{\text{amb}}) + L_{T1} \text{sgn}(T_{\text{meas}} - \hat{T}_1) \]  

where \( LT_1 > 0 \) is the tunable observer gain.

c) \( T \)-Observer 2:

\[ mc \dot{\hat{T}_2} = -hA(\hat{T}_2 - T_{\text{amb}}) + L_{T2} \text{sgn}(T_{\text{meas}} - \hat{T}_2) \]  

where \( LT_2 > 0 \) is the tunable observer gain.

2) Filters: Based on the error dynamics of \( V_c-Observer, T-Observer 1, \) and \( T-Observer 2 \) on the sliding surface, three filter expressions are derived. The outputs of these
filters are the residuals for fault detection, isolation, and estimation

\[ \dot{\hat{\vartheta}}_V = -\sqrt{\vartheta_T^2/(R + R_0)} \]  

(12)

where \( r_1 \) and \( \dot{\vartheta}_V \) are the equivalent output error injections from \( T - \text{Observer 1} \) and \( T - \text{Observer 2} \), respectively. The signals \( \dot{\vartheta}_T \) and \( \vartheta_T \) are continuous approximations (filtered versions) of the switching signals \( L_{\text{T1}}(T_{\text{meas}} - \hat{T}_1) \) and \( L_{\text{T2}}(T_{\text{meas}} - \hat{T}_2) \), respectively. The signals \( \dot{\vartheta}_T \) can be generated using the following equation: \( \dot{\vartheta}_T(s) = G_{\text{T1}}(s)U_{\text{T1}}(s) \), where \( \dot{\vartheta}_T(s) \) are the Laplace transforms of time-domain signals \( \dot{\vartheta}_V \) and \( L_{\text{T1}}(T_{\text{meas}} - \hat{T}_1) \), respectively, and \( G_{\text{T1}}(s) \) is a user-defined low-pass filter

\[ \dot{\vartheta}_V = -\sqrt{\vartheta_T^2}/(R + R_0) \]  

(13)

Next, we present the main result.

**Proposition 1:** Consider the system dynamics described by (1)–(5), the faulty sensor model described by (6), the observer structures (7)–(10), and the filter structures (11)–(13). If the observer gains satisfy the following conditions:

\[ L_V > \max(|f_1|_{\text{max}}, |f_2|_{\text{max}}) > 0 \]  

(14)

\[ L_{T1} > \max(|f_3|_{\text{max}}, |f_4|_{\text{max}}) > 0 \]  

(15)

\[ L_{T2} > |f_5|_{\text{max}} \geq |\dot{\vartheta}_V^2/(R + R_0) > 0 \]  

(16)

where \( |f_1|_{\text{max}} = (|\hat{\vartheta}_V|_{\text{max}})/|R_0C_0 \) + \( |\Delta V|_{\text{max}} \), \( |f_2|_{\text{max}} = (|\hat{\vartheta}_V|_{\text{max}})/|R_0C_0 \) + \( (|\Delta V^2|_{\text{max}}/|C_0|) + \Delta V_{\text{max}} \) with \( \Delta V_{\text{max}} = \Delta E_0 - \Delta I \), where \( \Delta E_0 \) is the result of the SOC error due to the use of faulty current measurement, \( |f_3|_{\text{max}} = |\Delta I_{\text{max}}^2 + 2|\Delta I_{\text{max}}|I_{\text{max}}/(R + R_0) \), \( |f_4|_{\text{max}} = |mc|\Delta I_{\text{max}} + hA|\Delta T_{\text{max}}|_{\text{max}} \), and \( |f_5|_{\text{max}} = |mc|\Delta T_{\text{max}} + hA|\Delta T_{\text{max}}|_{\text{max}} \); then defining \( r_1, r_2, \) and \( r_3 \) as the residuals, the sensor faults can be detected and isolated by the fault signatures given in Table I.

**Remark 1:** As mentioned in Table I, the estimates of the voltage, current, and temperature sensor faults can be given by \( r_1, r_2, \) and \( r_3 \), respectively.

**Proof:** We analyze three sensor fault cases separately.

1) **Case 1 (Occurrence of Voltage Sensor Fault Only):** As the fault is only in the voltage sensor, \( \Delta V \neq 0 \), whereas \( \Delta I = \Delta T = 0 \). Subtracting (7) from (1), the error dynamics of the \( V_c - \text{Observer} \) can be written as

\[ \dot{\hat{\vartheta}}_V = -\sqrt{\dot{\vartheta}_T^2}/(R + R_0) \]  

(17)

\[ \Rightarrow \dot{W}_1 = sV\hat{\vartheta}_V = sV(\hat{\vartheta}_V - \Delta V) \]  

(18)

and the error dynamics of the \( V_c - \text{Observer} \) can be written as

\[ \dot{\hat{\vartheta}}_V = \frac{\hat{V}_c - \hat{\vartheta}_V}{R_0C_0} - L_{\text{T1}}\Delta V \]  

(19)

Now applying the relationship \( \tilde{a}\tilde{b} \leq |\tilde{a}||\tilde{b}| \) on the first term of the right-hand side of (19), we have

\[ \frac{\dot{W}_1}{\sqrt{W_1}} \leq |\tilde{a}|(f_1 - L_v) \]  

(20)

where \( f_1 = -\sqrt{2}/(f_1|_{\text{max}} - L_v) > 0 \). For this case, the sliding surface \( sV = 0 \) will be achieved in some finite time

\[ t_1 \leq 2\sqrt{W_1(t_0)/\alpha_1} \]  

(21)

\[ \Rightarrow t_1 \leq 2\sqrt{W_1(t_0)/\alpha_1} \]  

(22)

\[ \dot{\hat{\vartheta}}_V = 0 \]  

(23)

where \( \vartheta_V \) is the equivalent output error injection signal defined after (11). As there is presence of fault \( \Delta V \),

<table>
<thead>
<tr>
<th>Residual Signals</th>
<th>Fault Detection/Isolation</th>
<th>Fault Estimate Signal</th>
</tr>
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<tbody>
<tr>
<td>( r_1 )</td>
<td>( r_2 )</td>
<td>( r_3 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
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<td>1</td>
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<td>1</td>
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</tr>
<tr>
<td>( \Delta V )</td>
<td>( \Delta I )</td>
<td>( \Delta T )</td>
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This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.
the left-hand side of (24) will be nonzero, thereby making \( \vartheta_V \neq 0 \). Now comparing (11) and (24) and considering a zero initial condition for \( \Delta V \) and \( r_1 \), it can be concluded that \( r_1 = \Delta V \), which proves that \( r_1 \) is an estimate of the voltage sensor fault. A similar analysis can be performed for \( T - \text{Observer 1} \) and \( T - \text{Observer 2} \) using the Lyapunov function candidates \( W_2 = 0.5mc_s^2 \) and \( W_3 = 0.5mc_s^2 \), respectively, where \( s_{TI} = T_{\text{meas}} - \tilde{T} \) with \( i = 1, 2 \). In this case, the observers act under nominal conditions since faulty voltage measurement does not affect the thermal dynamics, and therefore, we have \( r_2 = r_3 = 0 \). The observer gains should satisfy the conditions \( L_{T1} > 0 \) and \( L_{T2} > \left| \hat{V}_{C_{\text{max}}}^2 \right| (R + R_0) \) for error convergence.

2) Case 2 (Occurrence of Current Sensor Fault Only): In this case, we have only current sensor fault, and therefore, \( \Delta I \neq 0 \), whereas \( \Delta V = \Delta T = 0 \). Subtracting (7) from (1), the error dynamics of the \( V_c - \text{Observer} \) can be written as

\[
\dot{V}_c = -\frac{\dot{V}_c}{R_0C_0} - \frac{\Delta I}{C_0} - L_v \text{sgn}(V_c - r - \hat{V}_c) \tag{25}
\]

with \( \dot{V}_c = V_c - \hat{V}_c \) being the estimation error between the actual \( V_c = (E_0 - IRA) \) and the estimated \( V_c \). The sliding surface in this case is \( s_V = V_c - \hat{V}_c \). The sliding surface is therefore the estimation error between the actual and estimated \( V_c \). The sliding surface in this case is

\[
W_1 = \|s_V\|^2 \tag{26}
\]

\[
\Rightarrow \dot{W}_1 = 2s_V f_2 - L_v |s_V| \quad \text{with} \quad f_2 = -\frac{\dot{V}_c}{R_0C_0} - \frac{\Delta I}{C_0} + \Delta V_2. \tag{27}
\]

Now applying \( \dot{a} \leq |\dot{a}| \dot{b} \) on the right-hand side of (27), we have

\[
\dot{W}_1 \leq |s_V| (|f_2| - L_v). \tag{28}
\]

If the gain \( L_v \) is sufficiently high and positive, \( W_1 \leq 0 \) and can be written as

\[
\dot{W}_1 \leq -a_2 \sqrt{W_1} \quad \text{with} \quad a_2 = -\sqrt{2}(t \left| f_2 \right|_{\text{max}} - L_v) > 0
\]

\[
W_1(t) \leq \left( -\frac{a_2}{2} + \sqrt{W_1(t_0)} \right)^2. \tag{29}
\]

Note that the observer gain should satisfy the condition \( L_v > |f_2|_{\text{max}} = \left( \hat{V}_{C_{\text{max}}} / R_0C_0 \right) + (|\Delta I_{\text{max}}| / C_0) + |\Delta V_2|_{\text{max}} \), where \( |f_2|_{\text{max}} \) will be bounded under the assumption that \( \Delta V_2 \) and \( \Delta I \) are bounded by the finite values \( |\Delta V_2|_{\text{max}} \) and \( |\Delta I|_{\text{max}} \), respectively. Therefore, it can be concluded from the above analysis that the sliding surface \( s_V = 0 \) will be achieved in some finite time \( t_2 \leq 2 \sqrt{W_1(t_0) / a_2} \). On this sliding surface, we have \( s_V = 0 \) and \( \dot{s}_V = 0 \), which leads to

\[
\hat{V}_c + \Delta V_2 = 0 \tag{30}
\]

\[
\dot{\hat{V}}_c + \Delta V_2 = 0. \tag{31}
\]

Now, using (25) and (30), (31) can be written as

\[
\dot{V}_2 + \frac{\Delta V_2}{R_0C_0} - \frac{\Delta I}{C_0} = \vartheta_V \tag{32}
\]

where \( \vartheta_V \) is the equivalent output error injection signal defined after (11). Due to the presence of the fault \( \Delta I \), the left-hand side of (32) will be nonzero, thereby making \( \vartheta_V \neq 0 \). As \( \vartheta_V \neq 0 \), it can be concluded that \( r_1 \neq 0 \) from (11).

Subtracting (9) from (4), the error dynamics of the \( T - \text{Observer 1} \) can be written as

\[
mc_1 \dot{T}_1 = \{I^2 - (I + \Delta I)^2\}(R + R_0) - h \Delta \tilde{T}_1
\]

\[
- L_{T1} \text{sgn}(T_{\text{meas}} - \tilde{T}_1) \tag{33}
\]

with \( \dot{T}_1 = T - \tilde{T}_1 \) being the estimation error between the actual \( T \) and the estimated \( \tilde{T}_1 \). Note that the sliding surface in this case is \( s_{TI} = T_{\text{meas}} - \tilde{T}_1 = \tilde{T}_1 \), which is inside the sign term. The reachability condition for this sliding surface can be analyzed by choosing the Lyapunov function candidate \( W_2 = 0.5mc_s^2 \). Then

\[
W_2 = s_{TI} \left( \left| \Delta I \right|^2 - 2|\Delta I|I(R + R_0) \right).
\]

(34)

Now applying \( \dot{a} \leq |\dot{a}| \dot{b} \) on the first term of the right-hand side of (34), we have

\[
\dot{W}_2 = |f_3| |s_{TI}| - L_{T1}|s_{TI}| \tag{35}
\]

Now applying triangle inequality on the right-hand side of the above equation and then applying \( \dot{a} \leq |\dot{a}| \dot{b} \), we have

\[
\dot{W}_2 \leq |f_3| |s_{TI}| - L_{T1}|s_{TI}| \tag{36}
\]

With \( |f_3| = \left| \Delta I \right|^2 + 2|\Delta I|I(R + R_0) \),

For a sufficiently high choice of gain \( L_{T1} > 0 \), \( W_2 \leq 0 \) and can be written as

\[
W_2 \leq -a_3 \sqrt{W_2}, \quad a_3 = -\sqrt{2/mc}(|f_3|_{\text{max}} - L_{T1}) > 0
\]

\[
\Rightarrow \dot{W}_2(t) \leq \left( -\frac{a_3}{2} + \sqrt{W_2(t_0)} \right)^2. \tag{37}
\]

Note that the observer gain should satisfy the condition \( L_{T1} > |f_3|_{\text{max}} = \left| \Delta I \right|_{\text{max}}^2 + 2|\Delta I|I_{\text{max}}(R + R_0) \), where \( |f_3|_{\text{max}} \) will be bounded under the assumption that \( I_{\text{max}} \) is bounded and \( |\Delta I| \) is bounded by a finite value \( |\Delta I|_{\text{max}} \). It can be concluded from (35) that the sliding surface \( s_{TI} = 1 \) will be achieved in some finite time \( t_3 \leq 2 \sqrt{W_2(t_0) / a_3} \). On this sliding surface, we have \( s_{TI} = \tilde{T}_1 = 0 \) and \( \dot{s}_{TI} = \tilde{T}_1 = 0 \). Under this condition, from (30), we can write \( \vartheta_{T1} = (I^2 - (I + \Delta I)^2)(R + R_0) \), where \( \vartheta_{T1} \) is the equivalent output error injection signal defined after (13). Therefore, from (13), we can conclude that \( r_3 \neq 0 \).

For the error dynamics of the \( T - \text{Observer 2} \), a similar analysis can be performed using the Lyapunov function candidate \( W_3 = 0.5mc_s^2 \), where \( s_{T2} = T_{\text{meas}} - \tilde{T}_2 \). The observer gain should satisfy the condition \( L_{T2} > |f_2|_{\text{max}}(R + R_0) \) for the error convergence. Further, we can write \( \vartheta_{T2} = I^2(R + R_0) \), where \( \vartheta_{T2} \) is the equivalent output error injection signal defined after (13). However, in this case, we have \( \Delta I \neq 0 \), and therefore, we can conclude from (12) that \( r_2 \neq 0 \).
Moreover, \( r_2 \) also estimates the amplitude of the fault. Note that the sign of the fault can be estimated by evaluating the sign of \( \Delta V_2 \) using the filter (32).

3) Case 3 (Occurrence of Temperature Sensor Fault Only): Here, we consider a temperature sensor fault leading to the condition \( \Delta T \neq 0 \), whereas \( \Delta \eta = \Delta V = 0 \). In this case, the \( V_c - \) Observer acts under nominal conditions since faulty temperature measurement does not affect the voltage error dynamics, and therefore, we have \( r_1 = 0 \). The observer gain should satisfy the condition \( L_\nu > 0 \) for nominal error convergence. Subtracting (9) from (4), the error dynamics of the \( T - \) Observer 1 can be written as

\[
m \dot{\tilde{T}}_1 = -hA \tilde{T}_1 - L_{T1} \text{sgn}(T_{\text{meas}} - \tilde{T}_1)
\]

with \( \tilde{T}_1 = T - \tilde{T}_1 \) being the estimation error between the actual \( T \) and the estimated \( \tilde{T}_1 \). Note that the sliding surface in this case is \( s_{T1} = T_{\text{meas}} - \tilde{T}_1 = \tilde{T}_1 + \Delta T \), which is inside the sign term. The reachability condition for this sliding surface can be analyzed by choosing a Lyapunov function candidate \( W_2 = 0.5mc \tilde{T}^2 \). Then

\[
\dot{W}_2 = s_{T1}(mc \dot{\Delta}_T - hA \Delta_T + hA A_T - L_{T1} \text{sgn}(s_{T1}))
\]

\[
\Rightarrow \dot{W}_2 \leq (|f_{\Delta}| - L_{T1})|s_{T1}| \quad \text{with} \quad f_{\Delta} = mc \dot{\Delta}_T + hA A_T.
\]

Now, for a sufficiently high positive gain \( L_{T1} \), \( \dot{W}_2 \leq 0 \) and can be written as

\[
\dot{W}_2 \leq -a_4 \sqrt{W_2}, \quad \text{with} \quad a_4 = -\sqrt{2mc(|f_{\Delta}|_{\text{max}} - L_{T1})} > 0
\]

\[
\Rightarrow W_2(t) \leq \left[-\frac{a_4}{2}t + \sqrt{W_2(0)} \right]^2.
\]

Note that the observer gain should satisfy the condition \( L_{T1} > |f_{\Delta}|_{\text{max}} = mc |\dot{\Delta}_T|_{\text{max}} + hA |A_T x|_{\text{max}} \), where \( |f_{\Delta}|_{\text{max}} \) will be bounded under the assumption that \( \Delta_T \) and \( \dot{\Delta}_T \) are bounded by the finite values \( |\Delta_T|_{\text{max}} \) and \( |\dot{\Delta}_T|_{\text{max}} \). Therefore, from the above analysis, it can be concluded that the sliding surface \( s_{T1} = 0 \) will be achieved in some finite time \( t_4 \leq 2\sqrt{W_2(0)/a_4} \). In this sliding surface, we have \( s_{T1} = \tilde{T}_1 + \Delta_T = 0 \) and \( \dot{s}_{T1} = \dot{\tilde{T}}_1 + \dot{\Delta}_T = 0 \). Under this condition, from (36), we can write that

\[
mc \dot{\Delta}_T + hA A_T = \vartheta_{T1}
\]

(40)

where \( \vartheta_{T1} \) is the equivalent output error injection signal defined after (13). There is presence of fault \( \Delta_T \), the left-hand side of (40) will be nonzero, thereby making \( \vartheta_{T1} \neq 0 \).

Now comparing (13) and (40), it can be concluded that \( r_3 = \Delta_T \), which proves that \( r_3 \) is an estimate of the temperature sensor fault.

Subtracting (10) from (4), the error dynamics of the \( T - \) Observer 2 can be written as

\[
m \dot{\tilde{T}}_2 = I^2(R + R_0) - hA \tilde{T}_2 - L_{T2} \text{sgn}(T_{\text{meas}} - \tilde{T}_2)
\]

with \( \tilde{T}_2 = T - \tilde{T}_2 \) being the estimation error between the actual \( T \) and the estimated \( \tilde{T}_2 \). Note that the sliding surface in this case is \( s_{T2} = T_{\text{meas}} - \tilde{T}_2 = \tilde{T}_2 + \Delta_T \), which is inside the sign term. The reachability condition for this sliding surface can be analyzed by choosing a Lyapunov function candidate \( W_3 = 0.5mc \tilde{T}^2 \). Then

\[
\dot{W}_3 = s_{T2}(I^2(R + R_0) + mc \dot{\Delta}_T + hA A_T - hA s_{T2} - L_{T2} \text{sgn}(s_{T2}))
\]

\[
\Rightarrow \dot{W}_3 \leq (|f_{s}| - L_{T2})|s_{T2}| \quad \text{with} \quad f_{s} = I^2(R + R_0) + mc \dot{\Delta}_T + hA A_T.
\]

Now, for a sufficiently high positive gain \( L_{T2} \), \( \dot{W}_3 \leq 0 \) and can be written as

\[
\dot{W}_3 \leq -a_5 \sqrt{W_3}, \quad a_5 = -\sqrt{2mc(|f_{s}|_{\text{max}} - L_{T2})}
\]

\[
\Rightarrow W_3(t) \leq \left[-\frac{a_5}{2}t + \sqrt{W_3(0)} \right]^2.
\]

Note that the observer gain should satisfy the condition \( L_{T2} > |f_{s}|_{\text{max}} = mc |\dot{\Delta}_T|_{\text{max}} + hA |A_T x|_{\text{max}} + |I|_{\text{max}}(R + R_0) \), where \( |f_{s}|_{\text{max}} \) will be bounded under the assumption that \( \Delta_T \) and \( \dot{\Delta}_T \) and \( I \) are bounded, respectively, by the finite values \( |\Delta_T|_{\text{max}} \), \( |\dot{\Delta}_T|_{\text{max}} \), and \( |I|_{\text{max}} \). Therefore, from the above analysis, it can be concluded that the sliding surface \( s_{T2} = 0 \) will be achieved in some finite time \( t_5 \leq 2\sqrt{W_2(0)/a_5} \). In this sliding surface, we have \( s_{T2} = \tilde{T}_2 + \Delta_T = 0 \) and \( \dot{s}_{T2} = \dot{\tilde{T}}_2 = \dot{\Delta}_T = 0 \). Under this condition, from (41), we can write \( \vartheta_{T2} = mc \dot{\Delta}_T + hA A_T + I^2(R + R_0) \), where \( \vartheta_{T2} \) is the equivalent output error injection signal defined after (13). Therefore, from (12), we can conclude that \( r_2 \neq 0 \). \( \square \)

B. Effect of Modeling, Parametric, and Measurement Uncertainties

Although the battery system parameters are assumed to be constant at the design phase, in reality, they may vary due to temperature and current dependencies. Furthermore, there will be measurement and modeling uncertainties. In this section, we will analyze uncertainties affecting the proposed diagnostic scheme. With the inclusion of the additive uncertainties, the \( V_c \) dynamics (1) and the output (8) can be rewritten as

\[
\dot{V}_c = \frac{-V_c}{R_0 C_0} + L \eta_1 + \eta_2 \quad \text{for} \quad V_{c - \text{ref}} = E_0 - I_{\text{meas}} R - V_{\text{meas}} + \eta_2
\]

(45)

(46)

where \( \eta_1 \) and \( \eta_2 \) are the lumped effects of the uncertainties, which can be nonlinear functions of states and inputs, respectively. We assume that uncertainties and their derivatives are bounded, i.e., \( \eta_2, \eta_1, \dot{\eta}_1, \dot{\eta}_2 \in L_{\infty} \). Now, in the presence of no sensor faults, the error dynamics of the \( V_c - \) Observer can be rewritten as

\[
\dot{\hat{V}}_c = \frac{-\hat{V}_c}{R_0 C_0} - L \text{sgn}(\hat{V}_c + \eta_2) + \eta_1.
\]

(47)

Note that the sliding surface here is \( \hat{\nu}_V = \hat{V}_c + \eta_2 \). Now, even in the absence of any faults, the equivalent output error injection signal \( \hat{\nu}_V \neq 0 \) due to the presence of uncertainties. However, \( \hat{\nu}_V \) will be bounded with the assumption of \( \eta_2, \eta_1, \dot{\eta}_2 \in L_{\infty} \). The signal \( \hat{\nu}_V \) and therefore the residual \( r_1 \) will be nonzero even if there is no fault. Under these uncertainties, an upper bound on the residual \( r_1 \) can be computed as follows.
We assume that the observer gain $L_v > 0$ is sufficiently high to have reachability to the sliding surface. At the sliding surface, we have $\dot{s}_V = 0$ and $\dot{\nu} = 0$, which leads to: $\dot{V}_c + \eta_2 = 0$ and $\dot{V}_c + \dot{\eta}_2 = 0$. Therefore, (47) can be written as

$$\dot{\nu} = \frac{\eta_2}{R_0C_0} + \eta_1. \quad (48)$$

Substituting the above expression for $\dot{\nu}$ in (11)

$$\dot{r}_1 + \frac{r_1}{R_0C_0} = -\dot{\eta}_2 - \frac{\eta_2}{R_0C_0} - \eta_1 \quad (49)$$

$$\Rightarrow \frac{d(r_1 + \eta_2)}{dt} + \frac{(r_1 + \eta_2)}{R_0C_0} = -\eta_1. \quad (50)$$

From the solution of differential equation (50) and denoting $\alpha = 1/R_0C_0$, the time evolution of the signal $(r_1 + \eta_2)$ can be written as

$$r_1(t) = -\eta_2(t) + [r_1(0) + \eta_2(0)]e^{-\alpha t} - \int_0^t e^{-\alpha(t-\tau)}\eta_1(\tau)d\tau. \quad (51)$$

Using triangle inequality, the relationship $|\tilde{a}\tilde{b}| \leq |\tilde{a}||\tilde{b}|$, the uncertainty bounds $|\eta_1| \leq |\eta_1|_{\text{max}}$ and $|\eta_2| \leq |\eta_2|_{\text{max}}$ with $i \in \{1, 2\}$, and the fact that $e^{-\alpha t} > 0$, $\forall t \geq 0$, implies $|e^{-\alpha t}| = e^{-\alpha t}$, $\forall t \geq 0$, the upper bound of $r_1$ can be derived as

$$|r_1(t)| \leq |\eta_2|_{\text{max}} + \{|r_1(0)| + |\eta_2|_{\text{max}}\}e^{-\alpha t} + \int_0^t e^{-\alpha(t-\tau)}|\eta_1|_{\text{max}}d\tau. \quad (52)$$

Therefore, it can be concluded that the estimate of the voltage sensor fault using (11) will be corrupted by the effect of uncertainties.

**C. Threshold Design for Residuals: Passive Robustness to Uncertainties**

To suppress the effect of uncertainties, we design some nonzero constant threshold values against which the residuals will be compared. The residual evaluation logic will be as follows: residual $\geq$ threshold indicates fault and residual $< \text{threshold}$ indicates no fault. To design the threshold, we collect residual data under nonfaulty conditions either by Monte Carlo simulation or experimental studies. Then, we plot the probability distribution of the residuals. One example probability distribution is shown in Fig. 3. Note that this particular example probability distribution is generated based on the assumption of zero-mean Gaussian distribution of the uncertainties. In reality, this probability distribution will depend on uncertainties in the experimental data or of the Monte Carlo study. Then, we select a maximum allowable probability of false alarms.

From Fig. 3, it can be seen that the probability of the false alarms can be computed by the following equation:

$$P_{\text{FA}} = \int_{-\infty}^{-\text{th}} p_0(x)dx + \int_{\text{th}}^{+\infty} p_0(x)dx \quad (52)$$

where $\text{th}$ is the selected threshold and $p_0$ is the residual’s probability distribution under no fault. The goal here is to select $\text{th}$, which will give the acceptable $P_{\text{FA}}$ from (52).

**IV. SIMULATION AND EXPERIMENTAL RESULTS**

In this section, we test the effectiveness of the proposed diagnostic scheme by conducting simulation and experimental studies on a commercial A123 Li-ion battery cell. The studies are conducted on a constant parameter scenario, as discussed in Section III-A. First, using the experimental data, battery model parameters are extracted by fitting the battery model to the experimental data. This is done by solving an optimization problem, where a set of parameters that minimize the difference between experimental and model simulated data were identified. The identified model parameters are $R = 0.2$, $R_0 = 0.019$, $C_0 = 600$, $mc = 180$, $ha = 0.4$, and $E_0 = 2.939 + 0.01939 \times \text{SOC} - 0.000377 \times \text{SOC}^2 + 2.452 \times 10^{-6} \times \text{SOC}^3$.

**A. Simulation Studies**

Now, using the battery model identified in the previous step, we first perform simulation studies. To emulate a realistic scenario, measurement noise has been injected to the sensor outputs (standard deviation: 80-mA current noise, 50-mV voltage noise, and 0.5 °C temperature noise). To generate thresholds, the probability distribution of the nonfaulty residuals has been generated by a Monte Carlo study with 5% acceptable false alarm probability. For a cell-level diagnostic
Bias-type faults were injected to evaluate the performance of the scheme. The results are given in Figs. 5–7 for a voltage sensor bias fault of 0.1 V, a current sensor bias fault of 1 A, and a temperature sensor bias fault of 1 °C, respectively. Note that the diagnostic scheme is able to detect, isolate, and estimate the faults as per the fault signatures in Table I.

Next, we evaluate the performance of the diagnostic scheme in the presence of the parametric uncertainties. To this end, we examine the fault estimation errors and false alarm rates in such conditions. In the plant model, several important parameters \( R, R_0, C_0, hA, \) and \( Q \) are deviated from their nominal values to induce the uncertainties. The parameter deviations and corresponding estimation errors and false alarm rates are shown in Table II. It can be noted that the scheme significantly degrades under the deviation in the parameter \( R \), as evident in the large estimation errors and high false alarm rates. This is reasonable because the parameter \( R \) represents a large part of the battery internal resistance and hence is the most sensitive part of the electrical and thermal dynamics. Other than \( R \), the performance of the scheme is acceptable under the given deviations in the rest of the parameters.

**B. Experimental Studies**

In this brief, we show the effectiveness of the proposed scheme using experimental data collected from the physical battery. The experimental data along with the model data are shown in Fig. 8. To generate thresholds, the probability distribution of the nonfaulty residuals has been generated under different operating conditions by varying the input current up to 5C and different initial temperature conditions in 15 °C–40 °C range with 5% acceptable false alarm probability. Current profiles used in the study are constant discharge currents of 1C, 3C, and 5C and pulse discharge currents with pulse amplitudes of 1C and 4C. In the experimental data, we injected constant bias faults in the battery sensor outputs. The results given in Figs. 9–11 for a voltage sensor bias fault of 0.5 V, a current sensor fault of 2 A, and a temperature sensor bias fault of 2 °C, respectively, show a reasonable performance for the scheme. Note that in Fig. 11, the residual \( r_2 \) goes below the threshold toward the end. This is because the presence of uncertainty attenuates the fault effect in the residual. However, this should not affect the diagnosis as the residual was high...
Fig. 8. Current profile and corresponding voltage and temperature responses measured from the battery under no fault condition. The experimental voltage and temperature are compared with model outputs.

Fig. 9. Residual responses for a voltage sensor bias fault of 0.5 V (injected at 1150 s). The solid blue line represents the residuals and the red line represents the thresholds. Leftmost: the solid black line represents the injected fault and the blue line with x markers represents the $r_1$ residual.

Fig. 10. Residual responses for current sensor bias fault of 2 A (injected at 1150 s). The solid blue line represents the residuals and the red line represents the thresholds. Middle: the solid black line represents the injected fault and the blue line represents the $r_2$ residual.

Fig. 11. Residual responses for temperature sensor bias fault of 2 °C (injected at 1150 s). The solid blue line represents the residuals and the red line represents the thresholds. Rightmost: the solid black line represents the injected fault and the blue line with x markers represents the $r_3$ residual.

for a long period of time. To mitigate such effects, one might include the residual up time in the scheme to infer the presence of faults.

It can be seen that while the detection and isolation are successful, the estimates of the bias faults include errors due to uncertainties. In this study, we have found that the current, voltage, and temperature sensor fault estimation errors are within 3%, 10%, and 5%, respectively. Note that the filters (11)–(13) that are generating the residuals are stable. Therefore, as long as the uncertainties and their derivatives remain bounded, the fault estimates will be bounded around the neighborhood of the actual fault.

In the case of structured uncertainties that are far different from the fault in their frequency domain characteristics, it is possible to decouple the fault information from the uncertainties via appropriate filtering mechanisms. However, it is observed that in the case of batteries, the uncertainties are unstructured and can be close to or overlap with the frequency domain characteristics of the faults, which makes it very difficult to suppress their effects completely.

V. CONCLUSION

This brief outlined a diagnostic scheme for detecting, isolating, and estimating sensor faults in Li-ion batteries. The scheme uses three sliding mode observers based on the electrical and thermal dynamics of the battery. Further, some filter expressions have been derived using dynamics at the sliding motion, which are driven by the equivalent output injection errors from the sliding mode observers. The effect of modeling and parametric uncertainties on the diagnostic scheme has been analyzed. Finally, simulation and experimental studies have been conducted in a commercial A123 Li-ion battery cell to demonstrate the potential of the approach.

REFERENCES


