

HW 4: Optimal Economic Dispatch of Power Systems

Due: Friday July 7, 2017 at 17:00 CST

This assignment will provide hands-on practice for optimization with application to the economic dispatch problem in power systems. The assignment is organized in a tutorial fashion, thereby allowing you to practice optimization theory on a relevant real-world energy system example.

Reading

Read Sections 6.1 and 6.2 of Kirschen and Strbac's textbook "Fundamentals of Power System Economics," found in the "Reading" section of the course website.

[Optional] Read "Grid of the Future: Quantification of Benefits from Flexible Energy Resources in Scenarios With Extra-High Penetration of Renewable Energy", posted in the "Reading" section of the course website.

Background

Consider a power system with several generators supplying the required system load. Economic dispatch (ED) is the process of allocating generation levels to the individual generating units, such that the system load may be supplied entirely and most economically, subject to generator operating conditions and power system network constraints. This optimization task is a fundamental and daily operation of power systems. In this assignment, you will formulate and solve progressively more complex and realistic versions of the ED problem.

Consider the three-bus network power system in Fig. 1, which contains four generators, denoted A, B, C, D , and loads at each bus denoted by the downward arrows. Each branch has transmission capacity limits, provided in Table 1. Each generator has power capacity limits and a "marginal generation cost" provided in Table 2. "Marginal cost" implies the generation cost as a function of power is linear, and the marginal cost indicates the slope.

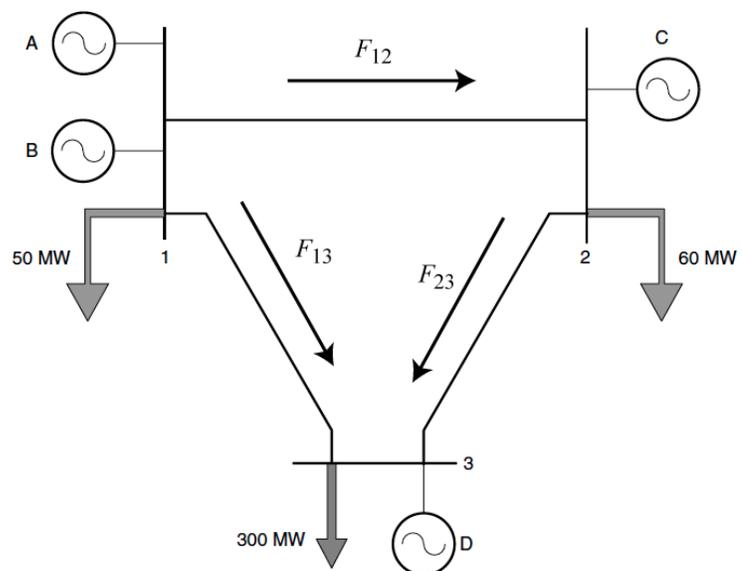


Figure 1: Three-bus network.

Table 1: Branch Data for Three-Bus System

Branch	Capacity [MW]	Reactance [p.u.]
1-2	126	$x_{12} = 0.2$
1-3	250	$x_{13} = 0.2$
2-3	90	$x_{23} = 0.1$

Table 2: Generator Data for Three-Bus System

Generator	Capacity [MW]	Marginal Cost [USD/MWh]
A	140	7.5
B	285	6
C	90	14
D	85	10

Problem 1: Intuition

Before formulating anything mathematically, use your intuition to determine how much each generator should supply to meet demand, while minimizing cost. Explain your solution, and the intuitive reasoning behind it. Do not spend too much time guessing. Your answer will undoubtedly be suboptimal. The point of this exercise is to compare your intuitive response with the optimal solution.

Problem 2: Formulation

In your report, formulate an optimization problem to minimize generating costs subject to supplying sufficient power to meet demand. Provide the following:

- (a) Define notation for the branch capacity, reactance, generator capacity, marginal cost, generator power, and bus demand. Define this notation within a table in your report.
- (b) What are the optimization variables?
- (c) Write down the objective function, using your notation from (a).
- (d) Write down ALL the constraints, using your notation from (a). Label the physical meaning of each constraint. For now, ignore the branch capacity constraints and reactance. This will be included later.
- (e) Is this a linear program (LP), quadratic program (QP), convex program (CP), or nonlinear program (NLP)? Why?

Problem 3: LP Solution

- (a) Encode this formulation into matrices c, A, b, A_{eq}, b_{eq} corresponding to the canonical LP problem $\min c^T x$, subject to $Ax \leq b, A_{eq}x = b_{eq}$. Write down matrices c, A, b, A_{eq}, b_{eq} in your report.
- (b) Solve the LP that you have formulated using CVX in Matlab or CVXPY in Python. Provide the minimum generation cost, i.e. the optimal objective function value. Provide the optimal generator powers, i.e. the minimizer. Qualitatively describe the optimal solution and compare to your guess. Which constraints are active?

Problem 4: Branch Capacity Calculation Next we incorporate the branch capacity constraints. To do this, we must calculate the branch power flows F_{12}, F_{13}, F_{23} shown in Fig. 1. Section 6.2.2 of Kirschen provides guidelines for computing these power flows by hand, using the principle of superposition.

Given the assumed flow directions shown in Fig. 1, we can write the power balance equation at each bus as follows:

$$\text{Bus 1 : } F_{12} + F_{13} = P_A + P_B - D_1 \quad (1)$$

$$\text{Bus 2 : } -F_{12} + F_{23} = P_C - D_2 \quad (2)$$

$$\text{Bus 3 : } -F_{13} - F_{23} = P_D - D_3 \quad (3)$$

where P_i represents the power generated from generator $i \in \{A, B, C, D\}$, and D_j represents the power demand at bus $j \in \{1, 2, 3\}$. In this case, we get three equations in three unknowns. However, these equations are linearly dependent since power balance (generation equals demand) holds for the entire system. For example, adding Equations (1) and (2) gives (3). Since one of these equations can be eliminated with no loss of information, we are left with two equations and three unknowns. This is hardly surprising because we have not taken into account the impedances (reactances in Table 1, in this case) of the branches. It turns out we can use Kirchoff's Voltage Law to create one more equation

$$x_{12}F_{12} + x_{23}F_{23} - x_{13}F_{13} = 0 \quad (4)$$

- (a) Re-write your LP formulation, but now include equality constraints (1)-(3) and (4). In addition, include inequality constraints that model branch capacity limits. How many optimization variables do we have now, and what are they? Hint: There are seven.
- (b) Augment your constraint matrices A, b, A_{eq}, b_{eq} from Problem 3 to include the branch capacity constraints. Solve the new LP in Matlab/Python. Provide the value of the objective function (total cost) and the value of the decision variables (generator powers) at the optimum. Describe qualitatively the optimal solution and compare to your guess. Which constraints are active?
- (c) Suppose the electric utility decides to invest into improving one of the three branches within the three-bus network. Which branch should they improve, and why? If they increased power capacity in this branch by 1 MW, then how much USD could be saved? Conversely, which branch (or branches) would yield no reduction in economic dispatch costs if it were improved?

Problem 5: Robust Economic Dispatch with Renewables. Next, we explore optimal economic dispatch with renewable generators. Consider Table 3. Generators C and D are now renewables, and have ZERO marginal cost for generation. However, their available capacity is uncertain and given by the ranges in Table 3. For example, imagine generator C is a wind farm, whose maximum output is dictated by environmental conditions. Similarly, imagine generator D is a solar farm, whose maximum output is dictated by environmental conditions. Our goal is to optimally schedule the generators, in the face of uncertain renewable capacity. Note, this problem closely follows Example 3.4 in the CH4 notes.

To start, we make the following helpful assumptions:

- Disregard line capacity constraints and reactance, just like Problems 2 and 3.

Table 3: Data with Renewable Generators

Generator	Capacity [MW], R_i	Marginal Cost [USD/MWh], c_i	Marginal Cost [USD/100%], f_i
A	140	7.5	1050
B	285	6	1710
C	[50 – 90]	0	0
D	[55 – 85]	0	0

- We redefine the optimization variables as $y \in \mathbb{R}^4$, $y_i \in [0, 1]$ which represents the generation percentage, where 100% represents generation at full capacity. For example, $y_A = 0.50$ schedules Generator A to produce 70 MW and $y_B = 1.00$ schedules Generator B to produce 285 MW.
- We redefine the marginal cost c_i as f_i , which represents the marginal cost of generation per 100% increase in generator output.
- We consider the aggregate demand to be $D_1 + D_2 + D_3 = 410$ MW.

(a) Formulate the optimal economic dispatch problem into a LP with format:

$$\text{minimize} \quad f^T y \tag{5}$$

$$\text{subject to} \quad a^T y \leq b \tag{6}$$

$$0 \leq y \leq 1 \tag{7}$$

Provide the values of vector f and scalar b . What does the vector a represent?

(b) We now hypothesize that vector a is uncertain, but lies within an ellipsoid

$$a \in \mathcal{E} = \{\bar{a} + Pu \mid \|u\|_2 \leq 1\} \tag{8}$$

Provide the values for \bar{a} and P . Hint #1: $\bar{a} \in \mathbb{R}^4$ represents the center of the ellipsoid. Hint #2: $P \succeq 0 \in \mathbb{R}^{4 \times 4}$ encodes the lengths of the semi-axes. If P is diagonal, then the diagonal elements represent the semi-axis lengths along each coordinate of a .

(c) The robust optimal economic dispatch problem is

$$\text{minimize} \quad f^T y \tag{9}$$

$$\text{subject to} \quad a^T y \leq b, \quad \forall a \in \mathcal{E} \tag{10}$$

$$0 \leq y \leq 1 \tag{11}$$

Convert this robust LP into a second-order cone program (SOCP). In your report, it is only necessary to provide the final written form of the SOCP.

- (d) Solve the SOCP in Matlab/Python. What are the optimal values of y ? What is the optimal cost? Interpret the result, as if you are explaining to a business executive why this result makes sense.
- (e) Suppose we use your SOCP result to schedule the four generators. Now let's perform some simple scenario analyses.
- Imagine the renewable capacities are realized to be $R_C = 70MW$ and $R_D = 70MW$. How much total power will be generated? Does this satisfy demand? Why or why not?
 - Imagine the renewable capacities are realized to be $R_C = 50MW$ and $R_D = 55MW$. How much total power will be generated? Does this satisfy demand? Why or why not?

Interesting Remarks

- Power system operators utilize day-ahead load predictions to solve the ED problem and procure generation on an hourly basis. Mismatch between predicted and actual load is compensated by “spinning reserves”.

Deliverables

Submit the following on bCourses. Be sure that the function files are named exactly as specified (including spelling and case). Please do NOT zip files.

LASTNAME_FIRSTNAME_HW4.PDF

LASTNAME_FIRSTNAME_HW4.xyz which contains your respective Matlab or Python files, i.e. $xyz \in \{m, ipynb\}$.

How to Download and Install CVX

Matlab Instructions

- Go to <http://cvxr.com/cvx/download/>
- In the “Download Matrix”, focus your attention on the “Standard bundles, including Gurobi and/or MOSEK”.
- Click the package corresponding to your system. For example, I have a MacBook Pro Early 2015, so I'll select `mexmaci64`. You can download a `.zip` or `.tar.gz` file.
- Download and unpack anywhere you like. The Downloads folder is a reasonable option.
- Start Matlab.
- Change directories to the top of the CVX distribution. Hint: Use command `» cd /Users/scottmoura/Downloads/cvx`

- Run Matlab command `> cvx_setup`
- The `cvx_setup` command runs a variety of checks to verify your installation is correct.
- After successfully installing CVX, please go to <http://cvxr.com/news/2014/02/cvx-demo-video/> to watch the Getting Started Demo from Prof. Stephen Boyd.
- To confirm a successful understanding, try coding and solving the example shown at <http://cvxr.com/cvx/>

Python Instructions

- Ensure you have installed Anaconda, as recommended in this class
- Go to <http://www.cvxpy.org/en/latest/install/index.html>
- Follow the instructions corresponding to your system
- After successfully installing CVXPY, fire-up the iPython notebook. Try running the example shown here: <http://www.cvxpy.org/en/latest/tutorial/intro/index.html>