Optimizing Energy Consumption in Caltrain’s Electric Distribution System
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Abstract
Caltrain is a Northern California commuter rail-line that will undergo a fleet replacement from diesel to electric-powered locomotives by 2019. The proposed traction power delivery system consists of a supply, distribution, and return system. This study focuses on the operation of the distribution system, which delivers traction power via overhead catenary wires that are supplied by a feeder system linked by seven paralleling stations. The amount of traction power required to operate the locomotive is referred to as traction load and is highly variable due to factors that affect the conductor’s operation of the locomotive. A challenge in operating the distribution system is to minimize the energy consumed by the paralleling stations given an expected power demand schedule for the locomotive engine. This study formulates an energy management problem that can be used to determine the optimal power generation schedule for the paralleling stations using a model scaled-down to a single locomotive and a short segment of the route.

(I) Introduction
(a) Motivation & Background
Caltrain’s modernization plan will replace its current diesel-powered locomotive fleet with an electric-powered fleet by 2019. The main motivation behind fleet electrification is to upgrade commuter rail service in response to rapid increases in ridership demand. In addition, electrification is expected to bring social, environmental, and economic benefits to the Bay Area (summarized in Table 1).

(b) Relevant Literature
Kneschke (2009) describes the proposed traction power system in great detail and analyzes the response of the Pacific Gas & Electric system to variable traction loads [1]. He provides a detailed schematic of the traction power system and data on traction load as a function of time of day.

Table 1
Summary of the expected benefits of Caltrain fleet electrification.

<table>
<thead>
<tr>
<th>Social</th>
<th>Environmental</th>
<th>Economic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Improved locomotive performance</td>
<td>1. Reduction in greenhouse gas emissions</td>
<td>1. Increased revenue due to increased ridership</td>
</tr>
<tr>
<td>a. Ability to accommodate rapid increase in ridership</td>
<td>2. Improvements in regional air quality</td>
<td>2. Reduction of fuel costs</td>
</tr>
<tr>
<td>b. Increased service, faster trips</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Decrease in traffic congestion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Reduction in engine noise</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Examples of challenges associated with managing this energy system include: short-circuiting and physical damage to the system, interactions with utility systems given load demand, and proper management to prevent equipment failure, and the effect of load fluctuation on the system. This study will focus on the last example – specifically, how load fluctuation along the route can affect the energy consumption of the distribution system.

My research background is in the measurement and dispersion modeling of black carbon emissions from Caltrain’s diesel locomotives. I worked on quantifying the emission rate as a function of power output, which required knowledge of power output as a function of how the operator drove the locomotive. I expect this knowledge to be useful in completing this project.
Schmedes et al. (2008) propose a design for the overhead catenary system to minimize the incidence of short circuiting [2]. They provide slightly more detail than [1] on the distribution system, and state that the paralleling stations are planned to be “6-7 miles apart”. This provides information on parameterizing the model that will be used in this study.

Hill (1994) provides a method to estimate traction load in railway traction power systems [3]. He provides a method to model vehicle dynamics and power output. This reference will help in creating a framework for the equations used to model the system.

Lu et al. (2010) uses dynamic programming to develop an optimal power management strategy for railroad vehicles [4]. Although their analysis focuses on diesel locomotives, the methods they employ in minimizing the fuel consumption of the engines can also be applied to power consumption in electric systems. This will provide a framework for the analysis used to optimize power generation.

(c) Focus of this Study

Caltrain’s proposed traction power delivery system consists of three parts: the supply system, distribution system, and return system. The goal of this project is to formulate an energy management problem (EMP) to minimize energy consumption within a short segment of the distribution system. A simple schematic of the system is shown in Figure 1.

The distribution system is composed of an overhead catenary system, a feeder system that links seven paralleling stations, and a switching station [1]. Traction load is highly variable and depends on how the operator drives the locomotive, which depends on the properties of Caltrain’s route (terrain, speed limits, number of stops, etc.)

This study considers a “toy” version of the entire system, where only a single train and a single segment of the route are modeled. The segment links two paralleling stations that combine to generate enough power for a given power demand. The methods used in this analysis can be scaled up to model the entire route.

Figure 1. A subset of the full proposed traction power system from Kneschke (2009). The shown segment shows three paralleling stations, one supply station, and a segment of the return system linking with the overhead catenary system that delivers power to the locomotives.

(II) Technical Description

(a) Model Description

A “toy” version of Caltrain’s electric system will be employed, which models one train traveling along a short segment of the route. The segment will span a distance of approximately 10 km between two paralleling stations, located at the Mountain View and Palo Alto Caltrain stations [2]. This stretch is one of Caltrain’s busiest – the Mountain View station ranks third in ridership, and the Palo Alto station ranks second [5]. Nearly all trains stop at both stations.

The San Antonio and California Ave stops are located in between the modeled segment. These two stations are stopped at less frequently, and are typically skipped in Caltrain’s express service trains. However, they need to be considered for local and limited service trains. The modeled train travels Northbound from the Mountain View station to the Palo Alto station, and three possible routes are considered: express trains that travel directly in between the two stations, limited service trains that stop at the California Ave station, and local service trains that stop at both the San Antonio and California.
Ave stations [7]. The start-and-stop driving cycle for the local and limited service trains will require a different power demand schedule than the express service trains, which typically cruise at the maximum speed.

(b) Model Specifications

The electric distribution system is modeled as two adjacent paralleling stations, a feeder system, and a catenary system. The paralleling stations contain autotransformers with 50 kV primary windings and 25 kV secondary windings. This allows for the transformation of the 50 kV voltage between the feeder and catenary to a 25 kV voltage between the catenary and rails [1]. As a result, the maximum possible voltage difference across the autotransformer and locomotive is 25 kV. The power capacity of the paralleling station autotransformers is 100 MW, which allows a maximum current of 4 kA to travel from each autotransformer to the distribution system.

Specifying the parameters of the wires used in the catenary system is necessary to determine the amount of power dissipated as current travels to the locomotive. To optimize distribution efficiency, the power dissipation should be minimized. The wire is copper with a cross-sectional area of 150 mm$^2$ [1], which corresponds to a resistance per unit length of 0.11 Ω km$^{-1}$.

It is necessary to define the mass and cross-sectional area of the locomotive to accurately model the vehicle dynamics. The locomotive has a mass of 70,000 kg and a cross-sectional area of 15 m$^2$. The passenger cars connected to the locomotive each have a mass of 45,000 kg and the same cross-sectional area as the locomotive. A train consists of one locomotive and three passenger cars, which gives a total mass of approximately 200,000 kg. Lastly, the locomotive has a maximum speed of 130 km h$^{-1}$ and a maximum power output of 4,000 horsepower, or 3.0 MW [6].

(c) Physical Model

The objective of the model is to minimize the energy consumption of the paralleling stations while still meeting the locomotive power demand. The total energy consumption (also the objective function) for the modeled segment is given as:

$$ J = \sum_{k=0}^{N-1} [P_{gen,1}(k) + P_{gen,2}(k)] \Delta t \tag{1} $$

Where:
- $J$ is the total energy consumption
- $\Delta t$ is the discrete time step
- $k$ is the indexing variable in time
- $N$ is the total number of time steps in the simulation
- $P_{gen,1}(k)$ is the power generated by the Mountain View paralleling station
- $P_{gen,2}(k)$ is the power generated by the Palo Alto paralleling station

Power generated by the paralleling stations is delivered to the locomotive’s motor via the catenary wires. The amount of dissipation in the wires depends on the resistivity of the wire and the length of wire conducting the current. Power conservation within the distribution system can be expressed as the following:

$$ P_{in}(k) = [P_{gen,1}(k) - r I_1(k)^2 x(k)] + [P_{gen,2}(k) - r I_2(k)^2 (d - x(k))]. \tag{2} $$

Where:
- $P_{in}(k)$ is the power delivered to the locomotive’s motor
- $I_1(k)$ is the current conducted by the wire segment connecting the locomotive to the Mountain View paralleling station
- $I_2(k)$ is the current conducted by the wire segment connecting the locomotive to the Palo Alto paralleling station
- $x(k)$ is the position of the locomotive, where $x = 0$ corresponds to the Mountain View paralleling station
- $d$ is the distance between the Mountain View and Palo Alto paralleling stations
- $r$ is the resistivity of the catenary wire.
Power demand is related to the power delivered to the locomotive’s motor by taking the motor efficiency into account:

\[ P_{\text{in}}(k) = \frac{1}{\eta(P_{\text{dem}}(k))} P_{\text{dem}}(k) \]  

(3)

Where:
- \( P_{\text{dem}}(k) \) is the power demand
- \( \eta(P_{\text{dem}}(k)) \) is the motor efficiency, which is a function of the power demand

The motor efficiency curve is roughly based off of typical efficiency curves for electric locomotives, where the efficiency is typically around 90% for a wide range of power outputs [3]:

\[
\eta(P_{\text{dem}}(k)) = \begin{cases} 
0.9 P_{\text{dem}}(k), & P_{\text{dem}}(k) \leq 0.1 P_{\text{max}} \\
0.1 P_{\text{max}}, & 0.9 \leq P_{\text{dem}}(k) \leq P_{\text{max}}
\end{cases}
\]  

(4)

The objective function can be rewritten as follows:

\[
J = \sum_{k=0}^{N-1} \left[ \frac{P_{\text{dem}}(k)}{\eta(P_{\text{dem}}(k))} + r l_1(k) x(k) + r l_2(k)^2 (d - x(k)) \right] \Delta t
\]  

(5)

\[ F(t) = \frac{P_{\text{dem}}(k)}{v(k)} \]  

(8)

Combining equations (6), (7), and (8) gives an equation describing train motion, known as Lomonosoff’s equation:

\[
M(a(k)) = F(k) - F_R(k) - M g \sin \alpha
\]  

(9)

Figure 2. A schematic of the electric distribution system.

To connect energy consumption to the movement of the vehicle, it is necessary to model the vehicle dynamics using Newton’s Second Law:

\[
M(a(k)) = F(k) - F_R(k) - M g \sin \alpha
\]  

(6)

Where:
- \( M \) is the mass of the vehicle (locomotive plus passenger cars)
- \( a(k) \) is the vehicle acceleration
- \( v(k) \) is the vehicle speed
- \( F(k) \) is the applied tractive force
- \( F_R(k) \) is the total resistive force
- \( g \) is the acceleration due to gravity
- \( \alpha \) is the slope angle

The total resistive force is a sum of three frictional forces that act on the vehicle, which include rolling friction, sliding friction, and aerodynamic drag. \( F_R(k) \) can be expressed as a function of \( v(k) \):

\[
F_R(t) = A + B v(k) + C v(k)^2
\]  

(7)

Where \( A, B, \) and \( C \) are empirical constants that can be estimated using physical constants and known vehicle parameters such as the mass and cross-sectional area [3].

The tractive force can be expressed in terms of the power demand, as follows:

\[
F(k) = \frac{P_{\text{dem}}(k)}{v(k)}
\]  

(8)

Figure 3. Free-body diagram detailing all forces acting on the locomotive.

It should be noted that this equation (9) only applies when the motor generates power that is diverted to the locomotive’s traction system. This occurs when the locomotive is accelerating, up
until it reaches its cruising speed. From that point onwards, the speed is defined beforehand in a similar fashion to the power demand schedule. Lastly, it is necessary to model the dynamics of the locomotive’s motor to connect the power demand to the current delivered through the system. This is simply done empirically: a given power demand corresponds to a certain engine rotation speed, which is used to calculate the torque. The torque is directly proportional to the current needed to drive the motor for the given power demand. The data in Table 2 (obtained from personal communication with Caltrain staff) is used to formulate a relationship between the power demand (in MW) and total current (in A):

\[
I_1(k) + I_2(k) = 1.44P_{\text{dem}}(k) + 1180
\]

Table 2
Empirical data used to derive the relationship between power demand and current.

<table>
<thead>
<tr>
<th>Power demand (kW)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>165</td>
<td>865</td>
</tr>
<tr>
<td>393</td>
<td>1834</td>
</tr>
<tr>
<td>705</td>
<td>2407</td>
</tr>
<tr>
<td>952</td>
<td>2766</td>
</tr>
<tr>
<td>1277</td>
<td>3133</td>
</tr>
<tr>
<td>1982</td>
<td>4289</td>
</tr>
<tr>
<td>2455</td>
<td>4700</td>
</tr>
<tr>
<td>2872</td>
<td>4984</td>
</tr>
</tbody>
</table>

(d) EMP Formulation
The objective of the EMP is to minimize energy consumption for the modeled segment of Caltrain’s route:

\[
\min_{x(k),u(k)} J = \sum_{k=0}^{N-1} \left[ \frac{p_{\text{dem}}(k)}{q(P_{\text{dem}}(k))} + rI_1(k)^2x(k) + rI_2(k)^2(d - x(k)) \right] \Delta t
\]

The states (denoted as \(X(k)\)) are the vehicle position, \(x(k)\), and speed, \(v(k)\), stepped forward in time as follows:

\[
x(k + 1) = x(k) + v(k)\Delta t
\]

\[
v(k + 1) = v(k) + a(k)\Delta t
\]

And subject to the following inequality constraints:

\[
0 \leq x(k) \leq d
\]

\[
0 \leq v(k) \leq v^{\max}
\]

Appropriate boundary conditions are also applied to the states:

\[
x(0) = 0
\]

\[
x(N) = d
\]

\[
v(0) = 0
\]

\[
v(N) = 0
\]

The currents (denoted in eqn. 11 as \(u(k)\)) are the currents through each segment of the catenary wire, \(I_1(k)\) and \(I_2(k)\). They are subject to equality constraints imposed in equations (9) and (10).

The currents are subject to the following inequality constraints:

\[
0 \leq I_1(k) \leq I^{\max}
\]

\[
0 \leq I_2(k) \leq I^{\max}
\]

(e) Program Formulation
Dynamic programming and the principle of optimality are the tools required to solve this EMP. The first step in formulating the program is to define the energy consumption per time step:

\[
g_k = \left[ \frac{p_{\text{dem}}(k)}{q(P_{\text{dem}}(k))} + rI_1(k)^2x(k) + rI_2(k)^2(d - x(k)) \right] \Delta t
\]

Next, the principle of optimality equation is formulated:

\[
V_k(x(k), v(k)) = \min_{I_1(k),I_2(k)} \left\{ g_k + V_{k+1}(x(k+1), v(k+1)) \right\}
\]
Where $V_k$ is the energy consumption from the $k^{th}$ time step to the end of the defined time horizon, $N$. The problem is solved backwards and recursively, starting with the boundary condition at the end of the time horizon:

$$V_N(x(N), v(N)) = 0$$  \hspace{1cm} (24)

This boundary condition states that no energy is consumed after the train reaches the end of the segment.

**(III) Results**

The program determines the optimal distribution of currents between the two segments of wire that connect the locomotive to each paralleling station. Intuitively, one should expect the distribution of current to shift from $I_1$ to $I_2$ as the locomotive travels along the segment. This maximizes the amount of current traveling through the shorter segment of wire, which reduces the total power dissipation.

Three different power demand schedules are considered: an express route where no stops are made in between the Mountain View and Palo Alto stations, a limited route where one stop is made at the California Ave station, and a local route where an additional stop is made at the San Antonio station for a total of two stops.

**(a) Express Route**

The power demand schedule for the express route starts with a linear increase in power demand. This occurs until the power output corresponding to the maximum speed is reached. After a while, the locomotive operator decreases the throttle while maintaining the same speed. The power demand decreases linearly back down to its idling value as the locomotive decelerates to a stop. As expected, the distribution of current shifts from $I_1$ to $I_2$ as the locomotive travels along the segment. However, due to a higher power demand at the beginning of the segment, the first paralleling station is required to generate more current (and thus power) than the second paralleling station. For the period of time where power demand plateaus at its maximum value of 3 MW, the first paralleling station generates power at its maximum capacity. This is an important consideration for the operation of this station, as overloading should be avoided at all costs.

**Figure 4.** Power demand and currents from each paralleling station plotted versus time for the express route case.

**(b) Limited Route**

The power demand schedule for the limited route starts in a similar fashion to the express route. However, the operator does not drive the locomotive at its maximum power output, stopping short of it at 2.6 MW. The locomotive cruises at a speed of 83 km h$^{-1}$ rather than its maximum speed of 130 km h$^{-1}$. It decelerates to a stop, idles for 90 seconds, and repeats the cycle until it reaches the end of the segment. Unlike for the express route case, overloading is a potential risk for both paralleling stations. The second plateau occurs towards the end of the segment, which requires the second paralleling station to generate more power in order to minimize power dissipation.
(IV) Discussion

Solving this energy management problem allows for a systematic method to optimally manage the power generated from the paralleling stations. The cases considered for a short segment can be scaled up to the entire route, forming a basis for a method to manage power generation schedules given highly variable power demand schedules. Furthermore, the parameters used in this study can be adjusted to fit any proposed electric rail system. As a locomotive fleets shift from diesel-powered to electric-powered engines, optimal energy management of the electric distribution system becomes an increasingly relevant problem.

(V) Summary

This study formulates and solves an energy management problem that optimizes power generation in Caltrain’s electric-rail distribution system in order to minimize total energy consumption. The system is modeled as a short segment along the entire route, where a single locomotive travels in between two paralleling stations that generate power to meet the locomotive’s power demand. An overhead catenary wire system delivers the generated current to the locomotive. The physical, first principles model of the system takes vehicle dynamics, power distribution, and locomotive motor operation into account. Dynamic programming and the principle of optimality are used to solve the problem backwards and recursively to determine an optimal distribution of current generated and distributed through the system.

The results and method of this study can be scaled up to formulate an optimal power generation schedule for all seven paralleling stations along Caltrain’s route. It can also be re-parameterized for existing electric rail-lines or in designing a proposed diesel-to-electric rail-line.
References


