LEC 16 : Combined Design & Control of a Fuel Cell Bus via Convex Programming

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Fuel Cell Hybrid Vehicles

AC Transit HyRoad Fuel Cell Bus

2016 Toyota Mirai
How Fuel Cells Work

Phosphoric Acid and P.E.M. Fuel Cells

Electron Flow

Load

Hydrogen

Anode

Electrolyte

Cathode

Hydrogen Ions

Oxygen

Water
**Figure:** Fuel cell hybrid powertrain. The vehicle is propelled by an electric machine (EM), which obtains energy from a fuel cell system (FCS), or an electric buffer (battery or supercapacitor). When EM operates as a generator, mechanical energy from the wheels is converted to (and stored as) electrical energy in the buffer.
Figure: Model of a bus line, expressed by demanded vehicle velocity and road altitude. The initial and final velocities and road altitudes, respectively, are equal, thus conserving kinetic and potential energy of the vehicle.
Research Question
Given a fixed bus line (i.e. velocity-road grade profile), optimize
- Fuel cell & super capacitor component sizes
- Energy management strategy for power-split
to minimize operating (hydrogen fuel) + component (FC + SC) costs

Unique Features
- Component sizes are static design variables (not time-varying)
- Energy management strategy is a multi-stage control process (time-varying)
Figure: Optimization framework for simultaneous component sizing and energy management of a hybrid city bus. After user inputs are provided, the combined operational and components cost are minimized simultaneously, in order to obtain the optimal power split control and sizes of powertrain components.
Optimization Problem

\[
\begin{align*}
\text{minimize} & \quad \text{Operation + Component Cost} \\
\text{subject to:} & \quad \text{Driving cycle constraints,} \\
& \quad \text{Energy conversion and balance constraints,} \\
& \quad \text{Buffer dynamics,} \\
& \quad \text{Physical limits of components,} \\
& \quad \ldots \\
& \quad (\text{For all time instances along the bus line}).
\end{align*}
\]
Useful Properties of Convex Function

- The function $f$ is said to be concave if $-f$ is convex.
- An affine function $h(x) = qx + r$ is both concave and convex.
- A quadratic function $f(x) = qx^2 + px + r$ with domain $f \in \mathbb{R}$ is convex if $p \geq 0$.
- A quadratic-over-linear function $f(x, y) = x^2/y$ with $\text{dom } f = \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$ is convex.
- The geometric mean $f(x, y) = \sqrt{xy}$ with $\text{dom } f = \{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0\}$ is concave.
- A nonnegative weighted sum $f = \sum w_if_i$, with $w_i \geq 0$, of convex functions $f_i$, is a convex function.
- A product $f(x, y) = xy$ is generally not a convex function.
Newton’s Second Law, Electric Machine Torque:

\[
T_{dem}(s_F, s_B, t) = \left( J_V + m(s_F, s_B) \frac{R_w^2}{r_{fg}^2} \right) \dot{\omega}_M(t) + \frac{\rho_{air} A_f c_d R_w^3}{2r_{fg}^3} \omega_M^2(t) \\
+ m(s_F, s_B) \frac{c_r R_w}{r_{fg}} g \cos \alpha(t) + m(s_F, s_B) \frac{R_w}{r_{fg}} g \sin \alpha(t)
\]

Vehicle Mass:

\[
m(s_F, s_B) = m_0 + s_F m_F + s_B m_B
\]
Electric Motor

\[ P_M(T_M, t) = a_0(\omega_M) + a_1(\omega_M) \cdot T_M(t) + a_2(\omega_M) \cdot T_M^2(t) \]

(a) Original model.

(b) Approximated model.
\[ P_{Ff}(P_{Fe}, S_F) = b_0 S_F + b_1 \cdot P_{Fe}(t) + b_2 \cdot \frac{P_{Fe}^2(t)}{S_F} \]

\[ 0 \leq P_{Fe}(t) \leq S_F P_{Fe,\text{max}} \]
Energy Storage Dynamics

\[ \dot{E}_B(t) = -P_B(t) \]

Resistive Losses

\[ P_{B,\text{loss}}(P_B(t), E_B(t)) = \frac{RC}{2} \frac{P_B^2(t)}{E_B(t)} \]

The SC energy level \( E_B(t) \) is related to the number of SC cells \( s_F n_0 \) and voltage \( V(t) \) according to

\[ E_B(t) = \frac{CV^2(t)}{2} s_B n_0. \]

Both pack energy, cell voltage, and electric current are limited according to

\[ 0 \leq E_B(t) \leq \frac{CV^2_{\text{max}}}{2} s_B n_0 \]

\[ i_{\text{min}} \sqrt{\frac{2n_0}{C} E_B(t)s_B} \leq P_B(t) \leq i_{\text{max}} \sqrt{\frac{2n_0}{C} E_B(t)s_B} \]

KEY STEP: Show these produce convex ineq. constraints, using convex fcn properties
Objective Function

Minimize operational cost (consumed hydrogen) and component costs (FC and SC)

\[ J(T_M(t), P_{Fe}(t), s_F, s_B, E_B(t), P_B(t)) = w_h \int_0^{t_f} P_{Ff}(P_{Fe}(t), s_F) dt + w_F s_F + w_B s_B \]
Objective Function

Minimize operational cost (consumed hydrogen) and component costs (FC and SC)

\[ J(T_M(t), P_{Fe}(t), s_F, s_B, E_B(t), P_B(t)) = w_h \int_0^{t_f} P_{Ff}(P_{Fe}(t), s_F) dt + w_F s_F + w_B s_B \]

subject to:
- Longitudinal Vehicle Dynamics
- Electric Motor Constraint Equations
- Fuel Cell Constraint Equations
- Supercapacity Constraint Equations
Objective Function

Minimize operational cost (consumed hydrogen) and component costs (FC and SC)

\[
J(T_M(t), P_{Fe}(t), s_F, s_B, E_B(t), P_B(t)) = w_h \int_0^{t_f} P_{Ff}(P_{Fe}(t), s_F) dt + w_F s_F + w_B s_B
\]

subject to:
- Longitudinal Vehicle Dynamics
- Electric Motor Constraint Equations
- Fuel Cell Constraint Equations
- Supercapacity Constraint Equations

Must show:
- Obj. Fcn. is convex w.r.t. design variables
- Inequality Fcns are all convex w.r.t. design variables
- Equality Fcns are all affine w.r.t. design variables
Table 5: Pseudo code in CVX for convex optimization of simultaneous component sizing and energy management of a hybrid city bus.

\[
\text{minimize } w_h \sum_{k=1}^{N} \left( b_0 s_F + b_1 P_{Fe}(k) + b_2 \frac{P_{Fe}^2(k)}{s_F} \right) \Delta t + w_f s_f + w_B s_B
\]

with respect to: \( P_{Fe}(k), P_B(k), E_B(k), T_M(k), s_F, s_B, \quad \forall k = 1, \ldots, N \)

subject to:
\[
T_M(k) \geq T_0(k) + T_1(k)s_F + T_2(k)s_B,
\]
\[
P_{Fe}(k) + P_B(k) - \frac{RC}{2} \frac{P_B^2(k)}{E_B(k)} - P_a \geq a_0(\omega_M(k)) + a_1(\omega_M(k))T_M(k) + a_2(\omega_M(k))T_M^2(k),
\]
\[
E_B(k + 1) - E_B(k) = -P_B(k)\Delta t,
\]
\[
E_B(N + 1) = E_B(1),
\]
\[
T_M(k) \geq T_{M\text{min}}(\omega_M(k)),
\]
\[
0 \leq P_{Fe}(k) \leq s_F P_{FeB\text{max}},
\]
\[
0 < E_B(k) \leq s_B \frac{C u_{max}^2 n_0}{2},
\]
\[
i_{min} \sqrt{\frac{2n_0}{C} E_B(k)s_B} \leq P_B(k) \leq i_{max} \sqrt{\frac{2n_0}{C} E_B(k)s_B},
\]
\[
s_f > 0,
\]
\[
s_B > 0,
\]
for all \( k = 1, \ldots, N \).
Optimal results for a FCHV city bus using supercapacitor as an energy buffer:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCS size</td>
<td>69.3 kW</td>
</tr>
<tr>
<td>Buffer size</td>
<td>0.7 kWh</td>
</tr>
<tr>
<td>Total cost</td>
<td>0.28 €/km</td>
</tr>
<tr>
<td>Computational time</td>
<td>&lt;10 s</td>
</tr>
</tbody>
</table>

Prices and bus specifications:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen price</td>
<td>4.44 €/kg</td>
</tr>
<tr>
<td>FCS price</td>
<td>34.78 €/kWh</td>
</tr>
<tr>
<td>Supercapacitor price</td>
<td>10 000 €/kWh</td>
</tr>
<tr>
<td>Yearly travel distance</td>
<td>70 000 km</td>
</tr>
<tr>
<td>Bus’ service period</td>
<td>2 years</td>
</tr>
<tr>
<td>Yearly interest rate</td>
<td>5 %</td>
</tr>
</tbody>
</table>

Optimal cost for different sizes of fuel cell system and electric buffer. The shaded region illustrates infeasible component sizes.
Further details in

Optimize component sizes (design) and energy management (control) of fuel cell city bus

Enormously complicated problem

THE SECRET: convex model formulation

Solutions in less than 10sec → rapid design iteration

This is just the beginning...
Case study 3: Plug-in hybrid electric vehicle (PHEV) in a series configuration

- Dual buffer consisting of Saft VL 45E battery and Maxwell BCAP2000 P270 supercapacitor.
- Can charge at 7 bus stops for 10 s, or 10 min before starting the route.

Engine generator unit (EGU).

Electric machine (EM).

Plug-in HEV powertrain in a series configuration. EGU = Engine generator unit, GEN = Generator.

Driving cycle with charging opportunities.
