Interactive Computation and Graphics of Simple Beams

ME 128 – Project 1

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Project: #1 Interactive Computation and Graphics of Simple Beams

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Introduction

The objective of this investigation is to design a software program to analyze a fixed/simply-supported steel wide flange I-beam under a concentrated load, concentrated moment, and distributed load based on theoretical principles. Results from the program are compared to hand derived calculations and finite element analysis in order to ascertain its accuracy. During each analysis method, simplifying assumptions are made and must be considered when comparing results. Subsequently, recommendations for a most successful and cost effective beam design are made.

The theory-based software program is written in MATLAB, utilizing its user-friendly graphics objects for input. The governing differential equations for elastic beam bending serve as the method of analysis. Derivations of all equations used are provided to the theory section of this report. Assumptions for this calculation include a two-dimensional beam modeled as a curve and theoretically perfect fixed/simple-supports.

The software used for finite element analysis (FEA) is SolidWorks with the built-in COSMOSWorks finite element method (FEM) analysis package. Three-dimensional models of each beam were drawn and placed under supports and loading conditions. Unlike the theoretical calculations, this analysis considers a fully three-dimensional beam with theoretically perfect fixed/simple supports.

The theoretically driven calculations and FEA provide reasonably similar results, particularly for beam displacement. However, stress concentrations in the FEA cause the maximum stress values to deviate from the theoretical values. This can be attributed to the different assumptions and methods of calculation used. Both of these methods suggest that a W6x9 beam will fail to meet the safety factor criteria for low-strength or high-strength steel. Alternatively, a W10x12 low-strength steel beam is recommended for its low price and ability to satisfy the safety factor requirement.

Nomenclature

Variables

A	Cross-sectional area of beam, [in^2]
c	Distance from neutral axis, [in]
C	Integration Constant
E	Young's modulus, [psi]
I	Moment of Intertia about x-axis, [in^4]
L	Beam length, [in]
M	Moment, [in-lb]
M_0	Concentrated Moment, [in-lb]
P	Concentrated load, [lbs]
q	Distributed load, [lb/in]
u	Deflection, [in]
V	Shear, [lbs]
w	Distributed load, [lb/in]
x	Position along length of beam, [in]
θ	Angle of deflection, [rad]
σ	Stress, [psi]

Subscripts

A	Fixed end of beam
B	Simply-supported end of beam
M_0	Concentrated Moment
P	Concentrated Force
w_1	Left-end of distributed load
w_2	Right-end of distributed load
x	x-direction
y	y-direction

Theory

The governing differential equations for elastic beam bending serve as the basis for the theoretical beam analysis. They are given below:

$$EI\frac{d^4u}{dx^4} = q(x) \qquad \text{Distributed Load} \qquad (1)$$

$$EI\frac{d^3u}{dx^3} = V(x) \qquad \text{Shear} \qquad (2)$$

$$EI\frac{d^2u}{dx^2} = M(x) \qquad \text{Moment} \qquad (3)$$

$$\frac{du}{dx} = \theta(x) \qquad \text{Angle of Deflection} \qquad (4)$$

$$u = u(x) \qquad \text{Deflection} \qquad (5)$$

These equations assume a perfectly elastic beam with small deflections. Additionally, the beam is modeled as a two-dimensional curve to simplify the derivations. For this project an elastic fixed/simply-supported beam under triple-state loading is considered. A schematic of the system is shown in Figure 1.

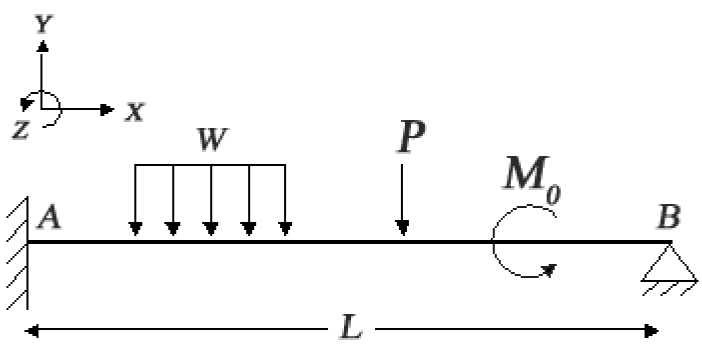

Figure 1: Elastic fixed/simply-supported beam under triple-state loading.

Since each load is governed by linear differential equations, each loading equation is superposed using the principle of superposition. This fact greatly simplifies the derivation process by considering only one load at a time. For illustration, the total displacement is calculated by summing each loading condition, $u_{total} = u_{distributed\ load} + u_{concentrated\ load} + u_{concentrated\ moment}$.

These equations make use of the Heaviside function, given by the expression within the angled brackets. The expression is evaluated when it is greater than zero. A separate MATLAB function was created for the purpose of evaluating this function within the program's beam calculations. It is shown in the Source Code.

Distributed Load

Governing Equations

$$EI\frac{d^4u}{dx^4} = w(x) = -w < x - x_{w_1} >^0 + w < x - x_{w_2} >^0 \qquad (6)$$

$$EI\frac{d^3u}{dx^3} = V(x) = -w < x - x_{w_1} >^1 + w < x - x_{w_2} >^1 + C_1 \qquad (7)$$

$$EI\frac{d^2u}{dx^2} = M(x) = -\frac{w}{2} < x - x_{w_1} >^2 + \frac{w}{2} < x - x_{w_2} >^2 + C_1 x + C_2 \qquad (8)$$

$$\frac{du}{dx} = \theta(x) = -\frac{w}{6EI} < x - x_{w_1} >^3 + \frac{w}{6EI} < x - x_{w_2} >^3 + \frac{C_1}{2EI}x^2 + \frac{C_2}{EI}x + C_3 \qquad (9)$$

$$u(x) = -\frac{w}{24EI} < x - x_{w_1} >^4 + \frac{w}{24EI} < x - x_{w_2} >^4 + \frac{C_1}{6EI}x^3 + \frac{C_2}{2EI}x^2 + C_3 x + C_4 \qquad (10)$$

Constants

$$C_1 = \frac{3w}{4L^3}\left[-\frac{1}{6}\left(L - x_{w_1}\right)^4 + \frac{1}{6}\left(L - x_{w_2}\right)^4 + L^2\left(L - x_{w_1}\right)^2 - L^2\left(L - x_{w_2}\right)^2\right] \qquad (11)$$

$$C_2 = \frac{w}{2}\left(L - x_{w_1}\right)^2 - \frac{w}{2}\left(L - x_{w_2}\right)^2 - C_1 L \qquad (12)$$

$$C_3 = 0 \qquad (13)$$

$$C_4 = 0 \qquad (14)$$

Concentrated Load

Governing Equations

$$EI\frac{d^4u}{dx^4} = w(x) = -P < x - x_P >^{-1} \qquad (15)$$

$$EI\frac{d^3u}{dx^3} = V(x) = -P < x - x_P >^0 + C_1 \qquad (16)$$

$$EI\frac{d^2u}{dx^2} = M(x) = -P < x - x_P >^1 + C_1 x + C_2 \qquad (17)$$

$$\frac{du}{dx} = \theta(x) = -\frac{P}{2EI} < x - x_P >^2 + \frac{C_1}{2EI} x^2 + \frac{C_2}{EI} x + C_3 \tag{18}$$

$$u(x) = -\frac{P}{6EI} < x - x_P >^3 + \frac{C_1}{6EI} x^3 + \frac{C_2}{2EI} x^2 + C_3 x + C_4 \tag{19}$$

Constants

$$C_1 = \frac{3P}{2L^3}\left[-\frac{1}{3}(L - x_P)^3 + L^2(L - x_P)\right] \tag{20}$$

$$C_2 = P(L - x_P) - C_1 L \tag{21}$$

$$C_3 = 0 \tag{22}$$

$$C_4 = 0 \tag{23}$$

Concentrated Moment

Governing Equations

$$EI\frac{d^4u}{dx^4} = w(x) = M_0 < x - x_{M_0} >^{-2} \tag{24}$$

$$EI\frac{d^3u}{dx^3} = V(x) = M_0 < x - x_{M_0} >^{-1} + C_1 \tag{25}$$

$$EI\frac{d^2u}{dx^2} = M(x) = M_0 < x - x_{M_0} >^{0} + C_1 x + C_2 \tag{26}$$

$$\frac{du}{dx} = \theta(x) = \frac{M_0}{EI} < x - x_{M_0} >^1 + \frac{C_1}{2EI} x^2 + \frac{C_2}{EI} x + C_3 \tag{27}$$

$$u(x) = \frac{M_0}{2EI} < x - x_{M_0} >^2 + \frac{C_1}{6EI} x^3 + \frac{C_2}{2EI} x^2 + C_3 x + C_4 \tag{28}$$

Constants

$$C_1 = \frac{3M_0}{2L^3}\left[(L - x_{M_0})^2 + L^2\right] \tag{29}$$

$$C_2 = -M_0 - C_1 L \tag{30}$$

$$C_3 = 0 \tag{31}$$

$$C_4 = 0 \tag{32}$$

The following boundary conditions were used to evaluate the constants of integration. The fixed/simply-supported ends of the beam prescribe these boundary conditions.

Boundary Conditions

$$y(x=0)=0 \tag{33}$$

$$\theta(x=0)=0 \tag{34}$$

$$y(x=L)=0 \tag{35}$$

$$M(x=L)=0 \tag{36}$$

Stress Calculations

Two theoretical stress calculations approximate the actual stress along the beam. These calculations operate under the simplification of a two-dimensional system. The normal stress is calculated via the flexure formula, as shown in Eqn. (37). The formula assumes pure beam bending and small deflections. It is shown later that the bending stress dominates, and is thus the primary factor to consider for failure. The second form of stress is the sheer stress, approximated by Eqn. (38). The norm of the normal and sheer stress is computed to evaluate the maximum stress along the beam. This value is then compared to the maximum stress using the Von Mises criteria for the FEA.

$$\sigma_x = \frac{M \cdot c}{I} \tag{37}$$

$$\sigma_y = \frac{V}{A} \tag{38}$$

Code Verification

The MATLAB program output for a W6x9 low-strength steel beam is compared to hand derived calculations to prove accuracy within the code. The results for each method at points A and B (the fixed and simply-supported ends, respectively) are given in Table 1. The values match perfectly, thus ensuring the program's precision.

Table 1: The accuracy of the code is verified by comparing the MATLAB calculations against hand derived calculations for a W6x9 low-strength steel beam. The computed values for each method are exactly the same, thus verifying the code's accuracy.

	u_A (in)	u_B (in)	θ_A (rad)	θ_B (rad)	M_A (in-lbs)	M_B (in-lbs)	V_A (lbs)	V_B (lbs)
MATLAB Calculations	0	0	0	0.009	-194669	0	5912.24	-4462.76
Hand Calculations	0	0	0	0.009	-194669	0	5912	-4463

Summary of Results

A program was developed to analyze the displacement, angle of displacement, moment, shear, and stress of a fixed/simply-supported beam under triple-state loading. These values are plotted against the beam length in the program's output. Additionally, maximum values and boundary values are output for each property. The maximum values are shown in Table 2. Examination of the W6x9 and W10x12 beams show that the latter experiences less than half of the maximum stress of the former.

Table 2: The MATLAB beam design analysis program provides the maximum deflection, angle of deflection, moment, sheer, and stress for the proposed beam. Note that the W10x12 beam experiences less than half the maximum stress of the originally proposed beam.

Beam Type	u_{max} (in) @ x (in)	θ_{max} (rad) @ x (in)	M_{max} (in-lbs) @ x (in)	V_{max} (lbs) @ x (in)	σ_{max} (psi) @ x (in)
W6x9	-0.31204 @ 70.4	0.0095798 @ 115.4	-194669 @ 0	5912.24 @ 0	35086 @ 0
W10x12	-0.09512 @ 70.4	0.0029202 @ 115.4	-194669 @ 0	5912.24	17934 @ 0

The W6x9 steel beam does not meet the prescribed safety factor of two, as determined by the design analyses performed in both MATLAB and SolidWorks. This result is true for both the low-strength and high-strength steel materials. As an alternative, a W10x12 low-strength steel beam is proposed. The additional cost incurred is only $37.50 more than a W6x9 low-strength beam.

Table 3: The originally proposed W6x9 beam does not meet the prescribed safety factor, regardless of material used. A W10x12 low-strength steel beam is proposed to satisfy the safety factor for an additional cost of $37.50.

Beam Type	Steel Strength	Total Cost	σ_{max} (psi) @ x (in)	Does Not Fail	Safety Factor Satisfied
W6x9	Low	$112.50	35086 @ 0	YES	NO
W6x9	High	$256.50	35086 @ 0	YES	NO
W10x12	Low	$150.00	17934 @ 0	YES	YES

Results

The theory-based software is designed using the MATLAB programming environment. Results are dynamically calculated based on the user's input. MATLAB offers the ability to utilize user-friendly graphics objects for ease-of-input. A screen shot of the MATLAB input dialog box is shown in Figure 2.

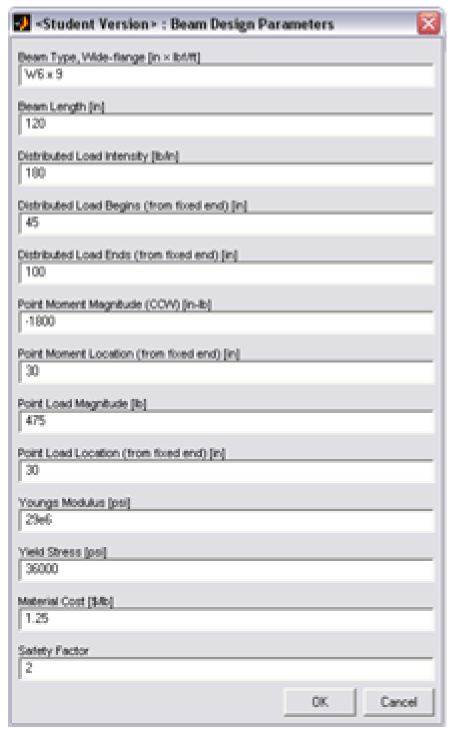

Figure 2: Input dialog box for theory-based MATLAB software.

The program results for a W6x9 low-strength steel beam are provided in Figure 3. These distributions are calculated dynamically based on the user's inputs.

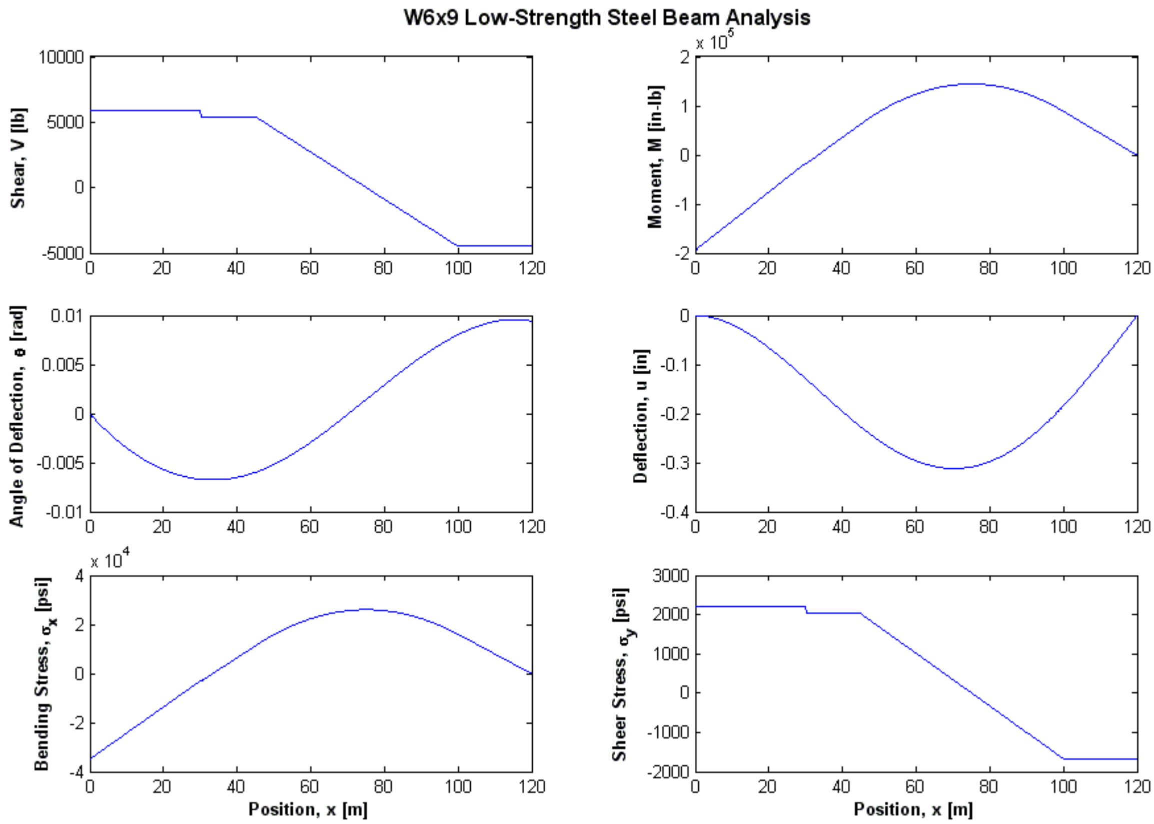

Figure 3: Shear, moment, angle of deflection, deflection, and stress vs. beam length for a W6x9 low-strength beam.

After displaying these distributions, the program informs the user if the proposed design will fail or does not satisfy the given safety factor. For the W6x9 beam, the program notifies the user that it does not meet the safety factor. Figure 4 shows an example of this message.

Figure 4: Example of program notification if the beam design does not satisfy the prescribed safety factor. This message appears when the initial W6x9 beam parameters are input into the program.

Additionally, a stress distribution plot is given for the top-half of the beam. Only the top half is given, because the distribution is symmetric across the neutral axis. Figure 5 shows an example for the W6x9 beam.

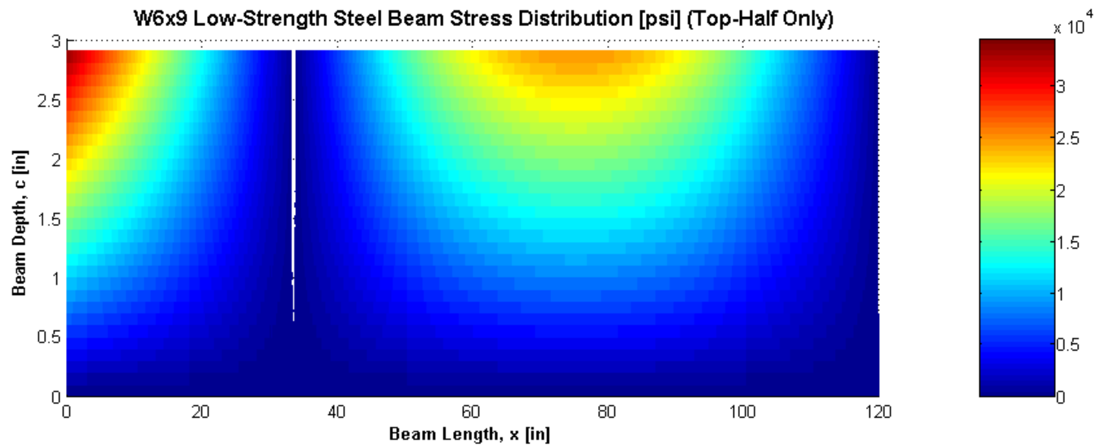

Figure 5: Stress distribution plot for the top-half of the W6x9 low-strength steel beam.

To satisfy the given safety factor, a W10x12 low-strength steel beam is proposed. This configuration was chosen based on its light weight and ability to meet the safety factor criteria. The loading distributions are provided in Figure 6.

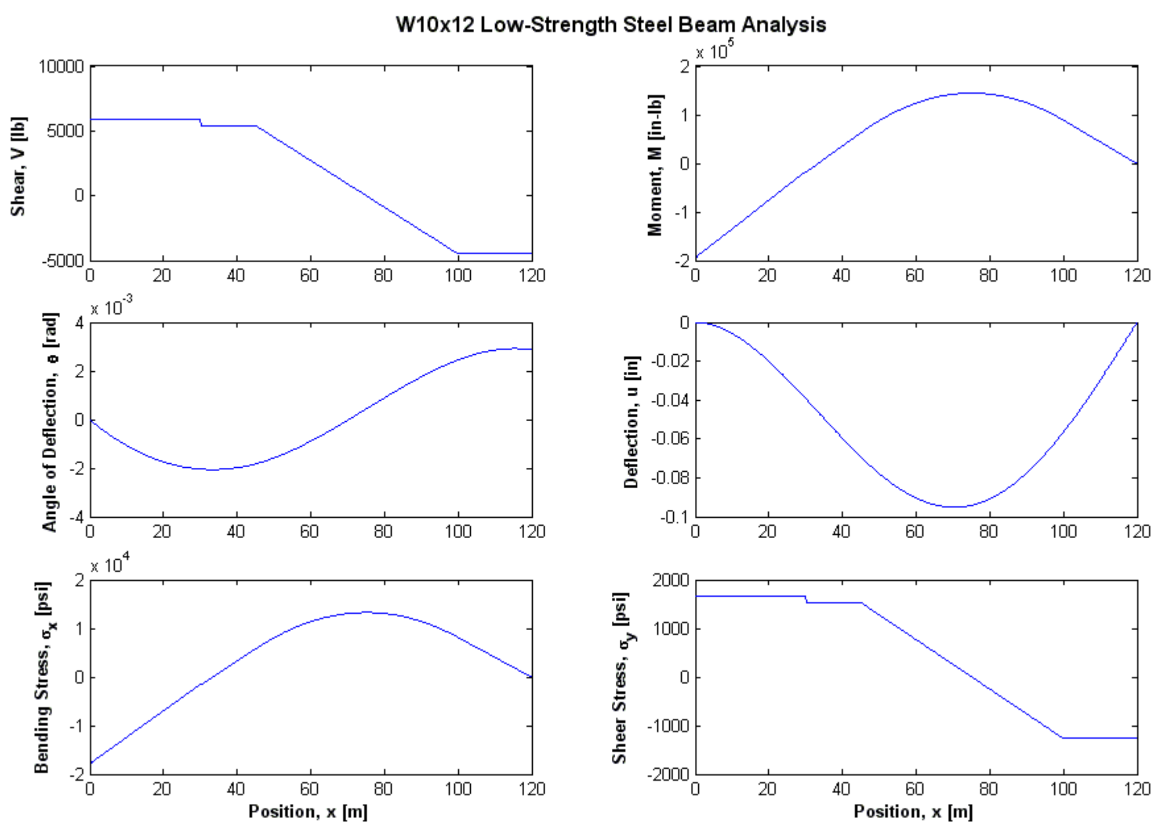

Figure 6: Shear, moment, angle of deflection, deflection, and stress vs. beam length for a W10x12 low-strength beam.

FEM Results

The finite element analysis was performed within SolidWorks using the COSMOSWorks package. This program allows the user to select a material for the beam from built-in libraries. Cast carbon steel is used in this analysis because it's Young's modulus and yield strength is nearly the exact same as the low-strength steel beam.

Various techniques are applied to simulate the fixed/simply-supported beam under triple-state loading (Figure 7). A fixed restraint is placed on the left face of the beam to eliminate all six degrees of freedom (DOFs). Creating a simple-support on the right side is more challenging. An immovable restraint is placed on the upper-right edge of the beam, allowing it to rotate, but not translate. It is important to note that the theory-based MATLAB program models the beam as a curve, whereas COSMOSWorks performs analysis on a fully three-dimensional beam. As such, the simple-support is modeled differently for each technique.

Since COSMOSWorks needs a point of reference for load placement, the beam is split into four sections. The concentrated load of 475 lbs. is located along the top edge that divides the first two sections. For the distributed load, 9900 lbs is applied over the entire top surface of the third section. This surface application is equivalent to a 180 lbs/in load across 55 in. The application of a point moment is slightly more complicated, but easily achieved. A force of 120 lbs. is placed 15 in. from the point moment location to create a resultant force couple.

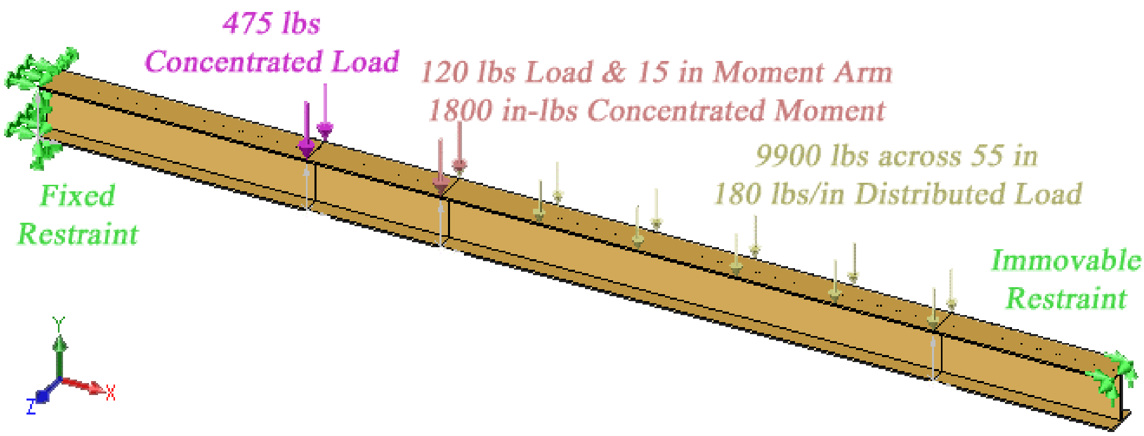

Figure 7: A fixed/simply-supported beam under triple-state loading is designed and analyzed in SolidWorks using COSMOSWorks. Various techniques are used to model the proposed two-dimensional system using a three-dimensional computer model.

Each beam type is studied using different meshing techniques. The first technique is a simple 0.7" mesh of 3D tetrahedral solid elements. Figures 8 and 9 show the displacement and stress distributions for the W6x9 cast carbon steel beam. Note the similar displacement range for the FEA calculations. On the other hand, the stress distribution range is much larger than the theoretical results.

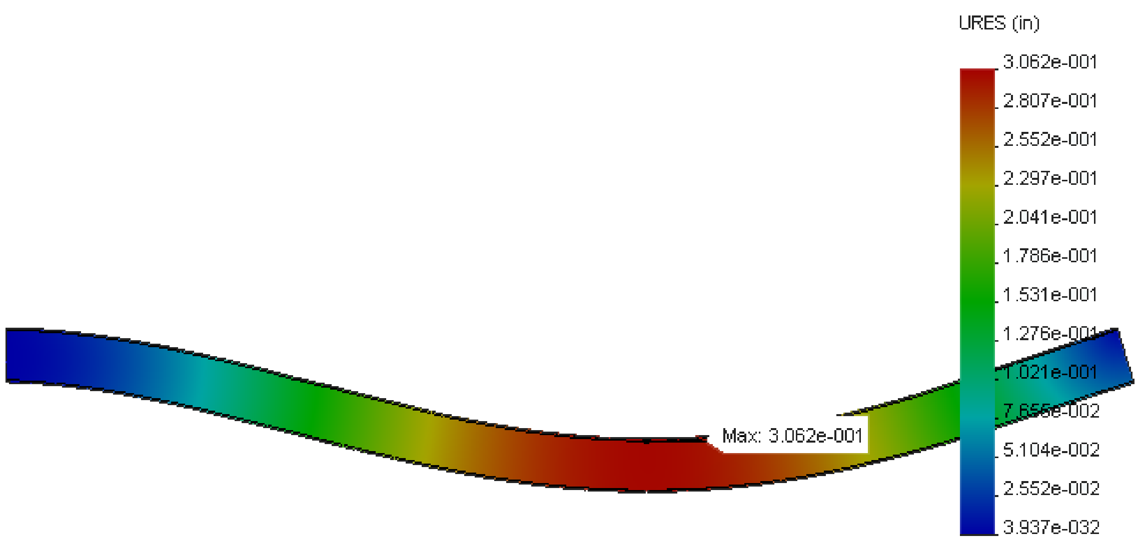

Figure 8: Displacement for a W6x9 cast carbon steel beam using a 0.7" mesh.

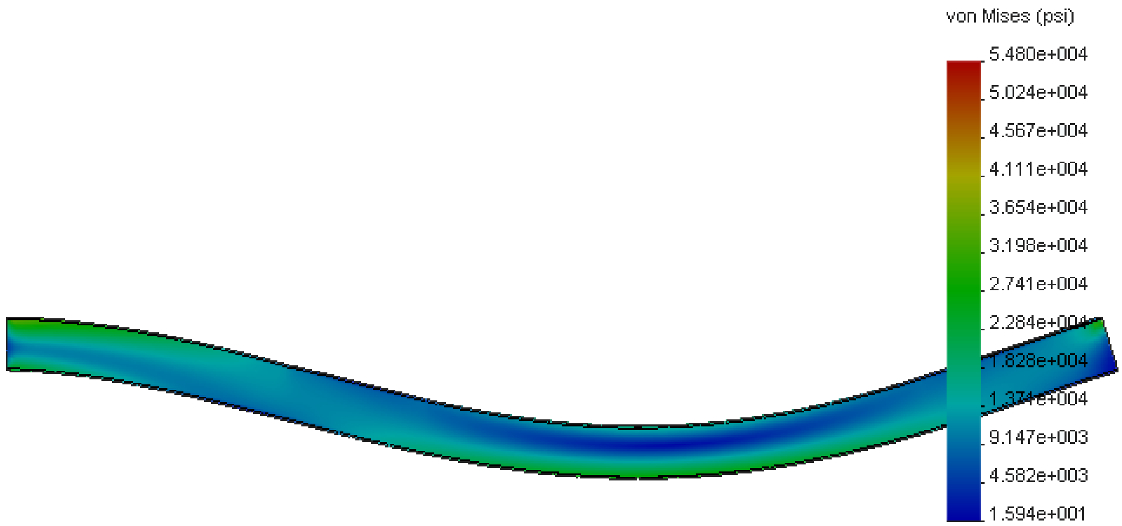

Figure 9: Stress distribution for a W6x9 cast carbon steel beam using a 0.7" mesh.

The second meshing technique uses the same 0.7" mesh, but adds a 0.35" control mesh at the supports. High accuracy calculations in these areas of maximum stress are preferable for detailed analysis. Figures 10 and 11 provide the FEA results for the W10x12 beam with the 0.35" control mesh. The maximum displacement is less than the W6x9 beam, yet the maximum stress is much greater.

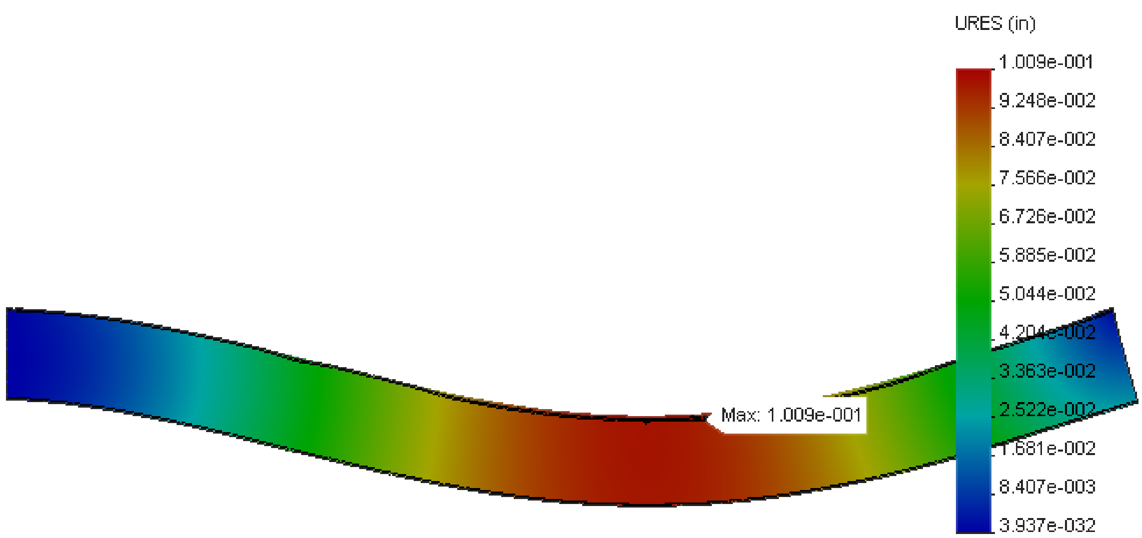

Figure 10: Displacement for a W10x12 cast carbon steel beam using a 0.7" mesh with 0.35" control mesh at ends.

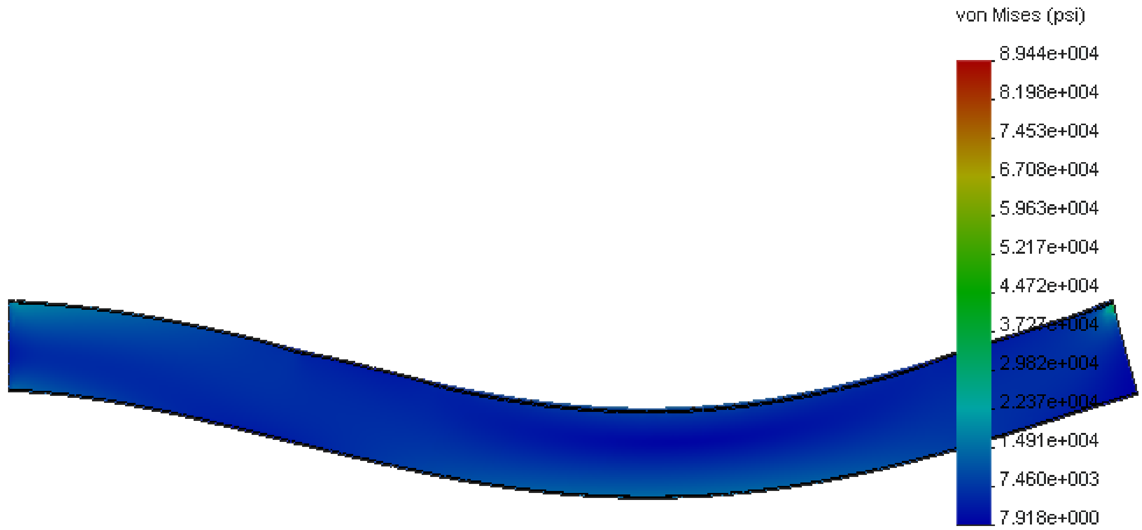

Figure 11: Stress distribution for a W10x12 cast carbon steel beam using a 0.7" mesh with 0.35" control mesh at ends.

Ideally, smaller and smaller meshes should be computed until the results do not vary significantly. However, smaller mesh sizes would cause the maximum stress to reach infinity for this system, since the reaction force at the simple support is acting on extremely small areas.

A comparison of the results from FEM and MATLAB are provided in Table 4. Both methods produce similar results for the maximum beam displacement. However, there is a huge discrepancy for the calculated maximum stress (nearly 400%). The FEA outcome can be contributed to the application of a force on a very small point at the simple-support. This result is discussed further in the conclusions section. Rather than providing non-comparable data, Table 4 provides the maximum stress, ignoring stress concentrations.

Table 4: FEM and MATLAB theoretical analysis comparison. The maximum displacements match well for each method. Conversely, the maximum stress does not due to stress concentrations. Maximum stress values provided here ignore those areas.

Beam Type	Analysis Method	u_{max} (in)	Difference w/ Theoretical (%)	σ_{max} (psi)	Difference w/ Theoretical (%)
W6x9	Theoretical	-0.31204	-	35086	-
W6x9	FEM – 0.7" mesh	-0.3062	1.87 %	33980	3.15%
W6x9	FEM – 0.7" mesh w/ 0.35" control at ends	-0.3064	1.81%	37580	7.1 %
W10x12	Theoretical	-0.09512	-	17934	-
W10x12	FEM – 0.7" mesh w/ 0.35" control at ends	-0.1009	6.08 %	21190	18.2 %

Interestingly, the difference between the theoretical and FEM values increase as the mesh size is decreased. This outcome can be attributed to the effect of stress concentrations at the supports.

Conclusions

The final results of the theoretical analysis and FEA show that a W6x9 beam would not satisfy the safety factor of two, using low-strength or high-strength steel. Therefore a low-strength W10x12 steel beam is recommended for its light weight and ability to satisfy the safety factor criteria. The additional cost is only $37.50 per beam. Other beam designs may exhibit better loading characteristics and satisfy the safety factor requirement. However, the W10x12 provides the optimum performance among the provided beam types. Further optimization can easily be achieved with larger sets of beam data, using the theory-based MATLAB program designed in this study.

Safety Factor

The originally proposed beam design fails to meet the safety factor criteria. Both methods of analysis agree on this fact. According to the theoretical analysis, the low-strength W6x9 steel beam has a safety factor of 1.03, and only 1.48 for the high-strength W6x9 steel beam. The FEA computations show that the original beam will easily fail at the supports. On the other hand, the proposed beam has a safety factor of 2.01 for low-strength steel. This design just nearly satisfies the safety factor criteria using theoretical analysis. For FEA, however, this beam also fails at the simple-support. However, this stress concentration must be ignored, since it is not realistic. In practice, no beam has a perfect simple support, but rather something more complicated. Additionally, no other portion of the beam witnesses a stress large enough to fail the safety factor requirement. As a result, it is the conclusion of this report that a W10x12 low-strength beam satisfies the prescribed safety factor.

Maximum Normal Stress

Steel wide-flange I-beams come in a variety of sizes that can be analyzed and optimized for this application. Since the normal bending stress is the dominant component, this value must be minimized. As shown in Eqn. (37), bending stress is proportional to the beam depth and inversely proportional to the moment of inertia. Therefore, it is desired to find a beam with the largest I / c ratio. Among all of the provided beam types, a W14x730 beam has the largest I / c ratio, at 127.56 compared to 5.5593 for the W6x9 beam. However, this beam would cost $9,125 per beam for low-strength steel and $20,805 per beam for high-strength steel. These prices are unreasonable for this simple application. Therefore, consideration for the total material cost is needed.

Total Material Cost

According to the project description, the total cost per I-beam is given by its weight. Consequently, it is a design goal to minimize the beam's weight. The standard steel wide-flange I-beam type is given in the form of *Waxb*, where *a* is the approximate beam depth and *b* is the beam's weight. The most cost effective design minimizes the value of *b.* As discussed before, the beam with the smallest maximum stress (W14x730) has an immense weight of 730 lbs compared to the original 9 lbs beam (W6x9). The cost of such a massive beam is surely beyond the means of most clients. The proposed W10x12 beam weighs only 3 lbs more than the original and just meets the safety criteria. The total cost of a low-strength W10x12 beam is $150, only $37.50 more than the $112.50

W6x9 beam. For that reason, the low-strength W10x12 beam is also the most cost-effective design.

Theoretical Analysis vs. FEA

As described in the results, the MATLAB program agrees very well with the hand derived calculations. Both methods also agree with FEA for displacement distributions. The values of maximum displacement differed from the theoretical values by less than 2% for the W6x9 beam and just over 6% for the W10x12 beam. These differences will decrease as the mesh sizes grow smaller. However, these aberrations are acceptable for basic analyses.

The theoretical analysis and FEA produce drastically different values for the maximum stress. This is the result of different assumptions used for each method. The differential equations do not account for stress concentrations at the boundaries and model a beam as a two-dimensional curve. In practice, this assumption is not accurate since an I-beam has important characteristics in all three dimensions. The FEA in COSMOSWorks models the simple-support by restraining the top-right edge of the beam from translation. This simulation creates a reaction force on an extremely small area, which does not make realistic sense. If this stress concentration is disregarded, the FEA does match the theoretical calculations to a certain extent.

Mesh Element Size

The global size for each 3D tetrahedral solid element has a direct effect on the accuracy of the FEM's results. As discussed in the results section, smaller mesh sizes will converge on values that are nearly identical to the theoretical calculations. For the purpose of basic analysis, a 0.7" mesh size is found to be adequate and reasonably quick for computation. COSMOSWorks also includes a useful feature to create smaller mesh sizes near areas of interest on the beam, thereby increasing accuracy. Control meshes of 0.35" are applied on the beam supports, where stress is at a theoretical maximum. This technique was not very useful at the fixed end because the values diverged as the mesh size grew smaller. Since FEA performs calculations at very small geometric scales, the smaller mesh increased the maximum stress at each support support.

Stress Concentrations

The theory-based differential equations do not account for the effects of stress concentrations at the supports. These effects are readily evident in the FEA. However, it must be acknowledged that the assumptions associated with the FEM model are unrealistic. Nonetheless, the effects of these numerically created stress concentrations are of great interest to this report.

The triple-state combined loading forces the top flange of the beam to bend downward near the simple-support. This downward force is resisted at the center of the flange, where it meets the web, as shown in Figure 12.

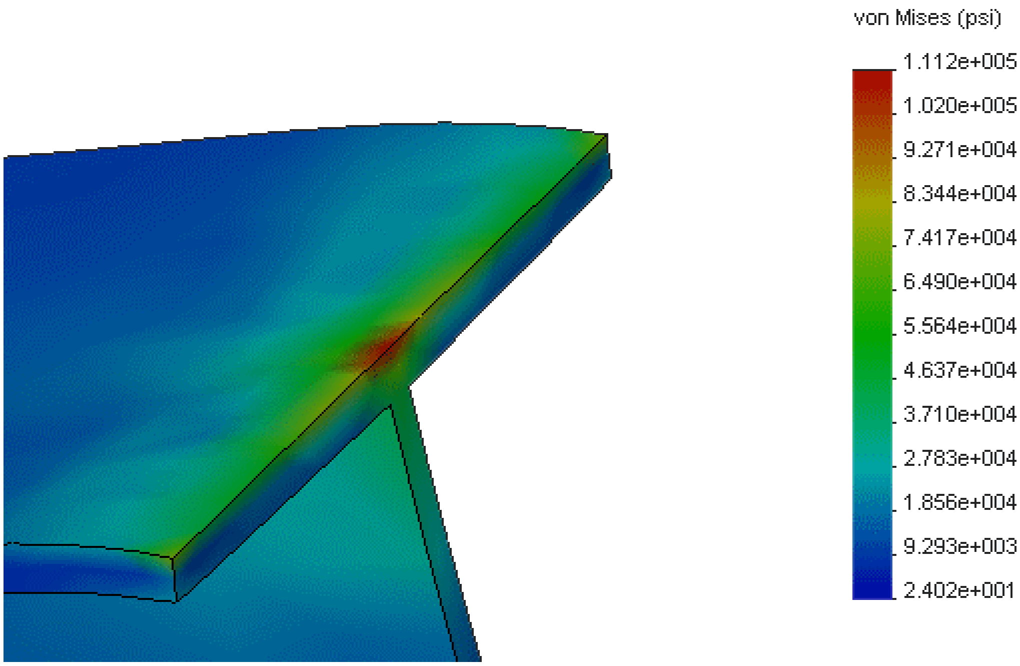

Figure 12: Stress concentration at the simple support of a W6x9 cast carbon steel beam using a 0.7" mesh with 0.35" control mesh at ends.

Table 5 lists the values of the stress concentrations at the simple-support. It is evident that smaller mesh sizes created by the controls creates larger stress values. This outcome is an effect of the numerical calculation performed on smaller geometries. As the mesh size grows smaller, the stress concentration will grow larger. Additionally, a much smaller and subtler stress concentration must occur at the fixed support. This fact would explain why stress increase as mesh sizes grow smaller.

Table 5: Stress concentration values at the simple-support for each beam type and mesh size.

Beam Type	Mesh Size	Simple-Support Stress Concentration (psi)
W6x9	0.7”	54,800
W6x9	0.7” mesh w/ 0.35” control at ends	111,200
W10x12	0.7” mesh w/ 0.35” control at ends	89,440

Although this region of high stress has a very interesting effect on the FEA results, it can be disregarded. In practice it is very unlikely that a beam would be supported by a theoretical simple-support, across an infinitely small area. As discussed previously, the restraints applied to the model analyzed in COSMOSWorks is accompanied by a variety of simplifying assumptions. This was one of them. To avoid these stress concentrations and provide more realistic results, SolidWorks models of the actual supports could be drawn and included in the FEA. However, that investigation is beyond the scope of this project.

Summary Conclusion

Through a theoretical and FEM analysis of a W6x9 fixed/simply-supported beam under triple-state loading, it is concluded that the beam falls short of the safety factor requirement. A low-strength W10x12 steel beam is recommended for an additional cost of $37.50. This beam meets the safety factor criteria while providing the most cost effective solution. This fact is proven theoretically and numerically using FEA, under the assumption that the supports are realistic and practical.

Source Code

beamdesign.m

```
%% Project #1 - Interactive Computation and Graphics of Simple
Beams
% Scott Moura
% SID 15905638
% ME 128, Prof. Lin
% Due Wed Feb 22, 2006

%% Nomenclature
% A             Cross-sectional Area
% c             Distance from Neutral Axis
% cost          Material Cost
% C             Integration Constant
% E             Young's Modulus
% I             Moment of Inertia
% L             Length of Beam
% M             Bending Moment
% M_0           Point Moment
% P             Point Load
% sigma         Bending Stress for 3D Distribution
% sigma_x       Bending Stress
% sigma_y       Shear Stress
% sigma_yield   Material Yield Stress
% theta         Angle of Deflection
% u             Beam Deflection
% V             Shear
% w             Distributed Load
% w_x           Distributed Load across the beam
% W             Weight
% x             Position, from left-end of beam
% x_M0          Location of Point Moment
% x_P           Locaiton of Point Load
% x_w1          Location of left-end of distributed load
% x_w2          Location of right-end of distributed load

clear

%% Input Beam Parameters
% GUI Input Window
prompt = {'Beam Type, Wide-flange [in × lbf/ft]','Beam Length
[in]','Distributed Load Intensity [lb/in]',...
          'Distributed Load Begins (from fixed end)
[in]','Distributed Load Ends (from fixed end) [in]',...
          'Point Moment Magnitude (CCW) [in-lb]','Point Moment
Location (from fixed end) [in]','Point Load Magnitude [lb]',...
          'Point Load Location (from fixed end) [in]','Youngs
Modulus [psi]','Yield Stress [psi]','Material Cost [$/lb]','Safety
Factor'};
dlg_title = 'Beam Design Parameters';
num_lines= 1;
def = {'W6 x 9','120','180','45','100','-
1800','30','475','30','29e6','36000','1.25','2'};
answer = inputdlg(prompt,dlg_title,num_lines,def);
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% Load Beam Data
[BeamData,BeamType] = xlsread('BeamData.xls');
BeamType = deblank(BeamType);       % Delete Trailing Blank Spaces
A = [];
for i = 1:length(BeamType)
    if strcmp(answer{1},BeamType{i})
        A = BeamData(i,1);
        c = (BeamData(i,2))/2;
        I = BeamData(i,6);
        tmpidx = findstr(BeamType{i},'x');
        W = str2num(BeamType{i}(tmpidx+2:end));
    end
end

if isempty(A)
    error('Invalid Beam Type')
end

% Set Variables
L = str2num(answer{2});
w = str2num(answer{3});
x_w1 = str2num(answer{4});
x_w2 = str2num(answer{5});
M_0 = str2num(answer{6});
x_M0 = str2num(answer{7});
P = str2num(answer{8});
x_P = str2num(answer{9});
E = str2num(answer{10});
sigma_yield = str2num(answer{11});
cost = str2num(answer{12});
f_S = str2num(answer{13});

%% Evaluate Theoretical Equations
% Initiallize Variables
x = 0:0.1:L;
C = zeros(4,3);
w_x = zeros(length(x),4);
V = zeros(length(x),4);
M = zeros(length(x),4);
theta = zeros(length(x),4);
u = zeros(length(x),4);

% Distributed Load Constants
C(1,1) = (3*w)/(4*L^3) * (-1/6*(L-x_w1)^4 + 1/6*(L-x_w2)^4 +
L^2*(L-x_w1)^2 - L^2*(L-x_w2)^2);
C(2,1) = w/2*(L-x_w1)^2 - w/2*(L-x_w2)^2 - C(1,1)*L;

% Point Load Constants
C(1,2) = (3*P)/(2*L^3) * (-1/3*(L-x_P)^3 + L^2*(L-x_P));
C(2,2) = P*(L-x_P) - C(1,2)*L;

% Point Moment Constants
C(1,3) = (3*M_0)/(2*L^3) * ((L-x_M0)^2 - L^2);
C(2,3) = -M_0 - C(1,3)*L;

% Calculate Beam Property Distributions
for i = 1:length(x)
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    % Distributed Load
    w_x(i,1) = -w * heaviside(x(i)-x_w1,0) + w * heaviside(x(i)-
x_w2,0);
    V(i,1) = -w * heaviside(x(i)-x_w1,1) + w * heaviside(x(i)-
x_w2,1) + C(1,1);
    M(i,1) = -w/2 * heaviside(x(i)-x_w1,2) + w/2 * heaviside(x(i)-
x_w2,2) + C(1,1)*x(i) + C(2,1);
    theta(i,1) = -w/(6*E*I) * heaviside(x(i)-x_w1,3) + w/(6*E*I) *
heaviside(x(i)-x_w2,2) + C(1,1)/(2*E*I)*(x(i))^2 +
C(2,1)/(E*I)*x(i) + C(3,1);
    u(i,1) = -w/(24*E*I) * heaviside(x(i)-x_w1,4) + w/(24*E*I) *
heaviside(x(i)-x_w2,4) + C(1,1)/(6*E*I)*(x(i))^3 +
C(2,1)/(2*E*I)*(x(i))^2 + C(3,1)*x(i) + C(4,1);

    % Point Force
    w_x(i,2) = -P * heaviside(x(i)-x_P,-1);
    V(i,2) = -P * heaviside(x(i)-x_P,0) + C(1,2);
    M(i,2) = -P * heaviside(x(i)-x_P,1) + C(1,2) * x(i) + C(2,2);
    theta(i,2) = -P/(2*E*I) * heaviside(x(i)-x_P,2) +
C(1,2)/(2*E*I) * (x(i))^2 + C(2,2)/(E*I)*x(i) + C(3,2);
    u(i,2) = -P/(6*E*I) * heaviside(x(i)-x_P,3) +
C(1,2)/(6*E*I)*(x(i))^3 + C(2,2)/(2*E*I)*(x(i))^2 + C(3,2)*x(i) +
C(4,2);

    % Point Moment
    w_x(i,3) = M_0 * heaviside(x(i)-x_M0,-2);
    V(i,3) = M_0 * heaviside(x(i)-x_M0,-1) + C(1,3);
    M(i,3) = M_0 * heaviside(x(i)-x_M0,0) + C(1,3)*x(i) + C(2,3);
    theta(i,3) = M_0/(E*I) * heaviside(x(i)-x_M0,1) +
C(1,3)/(2*E*I)*(x(i))^2 + C(2,3)/(E*I)*x(i) + C(3,3);
    u(i,3) = M_0/(2*E*I) * heaviside(x(i)-x_M0,2) +
C(1,3)/(6*E*I)*(x(i))^3 + C(2,3)/(2*E*I)*(x(i))^2 + C(3,3)*x(i) +
C(4,3);

end

% Distribution Totals (Superpose Equations)
w_x(:,4) = w_x(:,1) + w_x(:,2) + w_x(:,3);
V(:,4) = V(:,1) + V(:,2) + V(:,3);
M(:,4) = M(:,1) + M(:,2) + M(:,3);
theta(:,4) = theta(:,1) + theta(:,2) + theta(:,3);
u(:,4) = u(:,1) + u(:,2) + u(:,3);

% Stress Calculations
sigma_x = M(:,4)*c/I;
sigma_y = V(:,4)/A;
sigma = (sigma_x.^2 + sigma_y.^2).^0.5;
[sigma_max, tmpidx] = max(sigma);
x_sigma_max = x(tmpidx);

% Cost Calculations
TotalCost = cost * W * L / 12;

%% Analyze Results
% Calculate Maximum and Minimum Property Values
disp(answer{1})
disp('--------------------------------')
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```
[V_max, tmpidx] = max(abs(V(:,4)));
V_max = V(tmpidx,4);
x_V_max = x(tmpidx);

[M_max, tmpidx] = max(abs(M(:,4)));
M_max = M(tmpidx,4);
x_M_max = x(tmpidx);

[theta_max, tmpidx] = max(abs(theta(:,4)));
theta_max = theta(tmpidx,4);
x_theta_max = x(tmpidx);

[u_max, tmpidx] = max(abs(u(:,4)));
u_max = u(tmpidx,4);
x_u_max = x(tmpidx);

% Output Boundary Property Values
V_A = ['Sheer at A: ' num2str(V(1,4)) ' lbs'];
disp(V_A)
V_B = ['Sheer at B: ' num2str(V(end,4)) ' lbs'];
disp(V_B)
M_A = ['Moment at A: ' num2str(M(1,4)) ' in-lbs'];
disp(M_A)
M_B = ['Moment at B: ' num2str(M(end,4)) ' in-lbs'];
disp(M_B)
theta_A = ['Angle of Deflection at A: ' num2str(theta(1,4)) '
rad'];
disp(theta_A)
theta_B = ['Angle of Deflection at B: ' num2str(theta(end,4)) '
rad'];
disp(theta_B)
u_A = ['Deflection at A: ' num2str(u(1,4)) ' in'];
disp(u_A)
u_B = ['Deflection at B: ' num2str(u(end,4)) ' in'];
disp(u_B)
disp(' ')

% Output Maximum and Minimum Property Values
maxsheer = ['Maximum Sheer: ' num2str(V_max) ' lbs @ x = '
num2str(x_V_max) ' in'];
disp(maxsheer)
maxmoment = ['Maximum Moment: ' num2str(M_max) ' in-lbs @ x = '
num2str(x_M_max) ' in'];
disp(maxmoment)
maxangle = ['Maximum Angle of Deflection: ' num2str(theta_max) '
rad @ x = ' num2str(x_theta_max) ' in'];
disp(maxangle)
maxdeflect = ['Maximum Deflection: ' num2str(u_max) ' in @ x = '
num2str(x_u_max) ' in'];
disp(maxdeflect)
maxstress = ['Maximum Stress: ' num2str(sigma_max) ' psi @ x = '
num2str(x_sigma_max) ' in'];
disp(maxstress)
disp(' ')
dispcost = ['Total Cost: $' num2str(TotalCost)];
disp(dispcost)
```

```
% Create Plots
scrsz = get(0,'ScreenSize');
fig1 = figure(1);
%set(fig1,'Position',[scrsz(1) scrsz(2) scrsz(3) 0.9*scrsz(4)])

subplot(3,2,1)
plot(x,V(:,4))
ylabel('\bfShear, V [lb]')

subplot(3,2,2)
plot(x,M(:,4))
ylabel('\bfMoment, M [in-lb]')

subplot(3,2,3)
plot(x,theta(:,4))
ylabel('\bfAngle of Deflection, \theta [rad]')

subplot(3,2,4)
plot(x,u(:,4))
ylabel('\bfDeflection, u [in]')

subplot(3,2,5)
plot(x,sigma_x)
ylabel('\bfBending Stress, \sigma_x [psi]')
xlabel('\bfPosition, x [m]')

subplot(3,2,6)
plot(x,sigma_y)
ylabel('\bfSheer Stress, \sigma_y [psi]')
xlabel('\bfPosition, x [m]')

% Examine Failure Criteria
if (sigma_max > sigma_yield)
    warnstr = ['The maximum stress (' num2str(sigma_max) ' psi)
has exceeded the yield stress value (' num2str(sigma_yield) ' psi).
Please reconsider your beam design parameters.'];
    warndlg(warnstr,'Beam Failure')
elseif (sigma_max*f_S) > sigma_yield
    warnstr = ['The beam does not satisfy the prescribed safety
factor of ' num2str(f_S) '.  Please reconsider your beam design
parameters.'] ;
    warndlg(warnstr,'Safety Factor not satisfied')
end

%% Calculate 3D Stress Distribution
y = 0:0.1:c;
for i = 1:length(x)
    for j = 1:length(y)
        sigma_col(j,i) = abs(M(i,4)*y(j)/I);
    end
end

figure(2)
surf(x,y,sigma_col,'LineStyle','none')
colorbar
view(0,90)
xlabel('Beam Length, x [in]')
ylabel('Beam Depth, c [in]')
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heaviside.m

```
function y = heaviside(x,n)

% HEAVISIDE
% Returns x^n if x is positive and zero if x is negative

if (x > 0)
    if n >= 0
        y = x^n;
    else
        y = 0;
    end
else
    y = 0;
end
```